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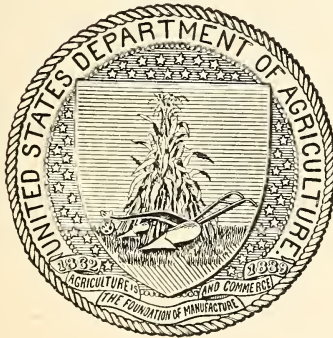
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MILTON WHITNEY, Chief.

HEAT TRANSFERENCE IN SOILS.

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LETTER OF TRANSMITTAL.

U. S. DEPARTMENT OF AGRICULTURE,
BUREAU OF SOILS,
Washington, D. C., March 5, 1909.

SIR: I have the honor to transmit the accompanying manuscript of an article entitled Heat Transference in Soils, by Harrison E. Patten, of this Bureau, and to recommend its publication as Bulletin No. 59.

Respectfully,

MILTON WHITNEY,
Chief of Bureau.

HON. JAMES WILSON,
Secretary of Agriculture.

PREFACE.

The dependence of so many of the physical properties of the soil upon the moisture content makes the study of the water-holding power of soils and distribution of water in soils of great theoretical and practical importance. With each and every soil there is a moisture content of exceptional interest, that is, where the water present is the maximum amount that can be held in the form of films over the surface of the soil grains and perhaps in the finest capillary spaces but is not sufficient to give any free water in the interstitial spaces. Practically, this amount of water can not be determined with any high degree of exactness, but all good greenhouse men, gardeners, etc., can approximate it for any given soil within a more or less narrow range of concentration. To them, this moisture content is known as the "optimum." Its significance has recently been discussed in bulletin No. 50 of this Bureau, and it has been shown that most if not all of the important physical properties of a soil show a critical change in passing through this optimum water content, from a lower to a higher content, or vice versa. This important moisture content, which is a characteristic constant for any particular soil, is in fact that at which the soil can be put in the best physical condition for plant growth; at which the most desirable "crumb structure" can be obtained; a maximum aeration and easiest penetration provided.

The primary object of the investigation described in the following pages was to determine the relation between the heat conductivity and moisture content of a soil with special reference to the importance of the "optimum water content." The analysis of the problem showed many experimental difficulties, which accounts for the meager progress hitherto made in the field. It has been shown that a soil does conduct heat better and more quickly with a moisture content near that recognized as the optimum, and the conditions which must be observed in future investigations on the relation of soil temperature to plant growth have been more clearly defined. The practical value of the work lies in pointing out the nature of the soil control which should be exercised in the planting of general farm crops to secure a warm seed bed and good germination, in the handling of cranberry marshes, and other special agricultural lines. By no means least is the value of this research in its relation to the theoretical viewpoint toward soil water which was developed in Bulletin No. 50, a viewpoint which has a rational experimental basis and is free from any arbitrary assumptions.

FRANK K. CAMERON,
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HEAT TRANSFERENCE IN SOILS.

INTRODUCTION.

Although measurement of soil temperature has been carried on in many countries for many decades, and possibly for many centuries, there seems to be no investigation on record giving the rate of flow of heat through soil under controlled laboratory conditions where the moisture content, packing (apparent specific volume), and effective specific heat are known. In some cases accurate daily readings of the soil temperature have been taken to a depth of 3 feet or more through a series of years, but no attempt made to measure, even roughly, the moisture content, the apparent specific volume, or the specific heat. Other observers have recognized the need of all these additional factors and have at times secured data on them under field conditions, but even then we can not calculate the heat conductivity of the soil with any degree of satisfaction from their readings, since the moisture content varies with the depth and we must take temperature readings at several depths to determine the temperature wave.

The investigators in this line of work have been interested mainly in ascertaining the temperature of the soil at given depths during the growing season. Some have tried the effect of various top dressings—manure admixtures, mulches, rolling, etc.—upon the soil temperature, and many of these results are valuable and reliable since they were gotten under comparable though undetermined conditions. Others, too, have recognized the part which the total quantity of available heat plays in the development of plants, and attempts to measure this heat have been made by integrating the air temperatures close to the ground and integrating the sunshine. Such calculations are also supplemented by an integration of the heat gained by the soil from day to day as the season progresses.

From another standpoint geologists and astronomers have made daily measurements of temperatures at regular depths in solid rock for periods even approaching half a century on one single series, and have accurately calculated the heat conductivity of the rock. But such values for the heat conductivity of a homogeneous or compact

rock conglomerate do not help much in estimating the conductivity of a soil.

Much time has been devoted to the laboratory study of the heat relations of soils, such as heat capacity and the effect of soil moisture upon it; demonstration of the additive property for the heat capacity of a powder consisting of different minerals each with its own specific heat; the effect of top dressings and admixtures, etc. Especially interesting is the laboratory work of Petit^a upon the penetration of frost into soils. He found that the phenomenon of supercooling, such as is observed in the freezing of water in the form of drops or in capillary tubes, also occurs in the freezing of soil water, and may be as great as 3° C. below the freezing point of water. The temperature to which soil water can be supercooled appears to be lower the lower the moisture content of the soil and the greater the energy with which the water is held by the soil. The passage of frost into the ground is fastest for quartz sand, slower for clay, and slowest for humus (peat). For continued frost the soil temperature sinks after freezing, the faster and the deeper the lower the moisture content of the soil, and conversely for the thawing of a soil. Living or dead vegetation or snow protects from frost and retards thawing.

The heat conductivities of dry powders and soils have been determined by numerous experimenters (see Tables XV, XVI, and XVII, but the effect of moisture upon the heat conductivity has been treated only qualitatively. Consequently an attempt is made in this paper to establish a quantitative relation between the heat conductivity of a powder and its moisture content. Since the apparent specific volume and effective specific heat of a powder are factors in determining its heat conductivity, these, too, were controlled. The diffusivity of the powder, of course, was found in order to calculate the heat conductivity.

The importance of such an investigation is apparent upon reading the reports of experiment stations generally, both in this country and abroad, since large sums of money and patient observation and experimentation are involved in the attempt to ascertain the relation of temperature in air and in soil to crop growth. So long as the factors which control heat conduction remain in doubt as to magnitude and direction of influence much of this field work upon soil temperatures will be in vain.

Usually the heat conductivity of the earth's crust has been determined by taking the temperature of the earth at different depths as affected by the alternate heating and cooling at the surface, due to the change from night to day, and in a greater degree as affected by the progression of the seasons.

^aForsch. Geb. Agr. Phys., 16, 285 (1893).

Such measurements give the temperature wave at each depth very well for the particular locality studied, and when the specific heat of the dry soil, the per cent of water, and the specific volume (or packing) are known, one may calculate the heat conductivity of the soil. This calculation, however, involves the treatment of a periodic function which only approximates a *sine curve*, since it is modified by a damping term as well as by other disturbing factors. Consequently, the heat conductivity calculated in this manner is at best an approximation. Further, as the heat wave penetrates into the earth, distillation of moisture takes place accompanied by a change in packing, so any method of measuring heat conductivity which requires the soil to be subject to a difference of temperature for a short period can not be expected to give even approximately accurate results.

The selection of a method for measuring the passage of heat through a moist soil under laboratory conditions is a case of having to choose the least of several evils. There is no good method. A few of the experimental difficulties are discussed below. Briefly, the situation is this: If the heat be measured by extracting it from the soil, distillation ensues and the soil-crumbs structure is altered; if heat be added, the same process is gone through in the reverse sense; if alternate heat and cold be applied, the soil changes its packing and may change its moisture content. In the face of these conditions it was decided to supply heat, from an infinite source at a constant temperature, to the end of a soil column and take the rate of rise of thermometers at regular distances down the column, and base a calculation of the heat conductivity upon the behavior of that portion of the soil which was sufficiently removed from the source of heat to be very little changed by distillation of its moisture and yet close enough to show the influence of the heat. There are grave defects in this method, as shown in the discussion of sources of error below, but it has served to give a fairly quantitative idea of the effect of soil moisture upon the heat conductivity of soils.

METHOD.

In general, when a column of material is heated at one end the temperature rises gradually all along the column. If the heated end be maintained at a constant temperature, each cross section of the column will finally attain a steady temperature and transmit all of the heat it receives. Thus let one end of the bar in figure 1 be kept at 100°C . and the other at 0°C . Then after a long time the temperature at any given point along the bar will be proportional to its distance from the end. If, for instance, the bar be exactly 100 cm. long, its temperature will drop at the rate of 1 degree for every centimeter passing from the heated end.

From the fact that $\frac{d^2\theta}{dx^2}$ is zero when the temperature gradient is constant along the bar, it follows that in the steady state this excess of heat likewise is zero, provided the surface of the bar is insulated, thus preventing radiation of heat from the edges, whose depth is Δx of the cross section A . That is to say, as much heat flows out of the surface of cross section A at $x + \Delta x$ as enters the surface at x .

For the case where the surface is not insulated,^a

$$KA \left(\frac{d^2\theta}{dx^2} \right) \Delta x = Ep \Delta x \theta,$$

where E is the surface emissivity of the bar and p is the perimeter of the element, $p \Delta x$ being the surface of the element of cross section A , and width Δx which is radiating heat.

Forbes's^b method of determining heat conductivity avoids the solution of Fourier's equation and the hypotheses (a) of constant heat conductivity, and (b) of Newton's law of cooling, upon which Fourier's equation is founded. Forbes determines the heat conductivity directly in terms of the quantities which enter into its definition.

Forbes's method involves two separate experimental procedures:

(a) The *statical experiment*, which consists in maintaining one end of a long bar at a constant temperature until the temperature at each point along the bar has ceased to change. Thus the temperature gradient along the bar is determined.

(b) The *dynamical experiment*. An iron bar similar in all respects to that employed in the statical experiment, but much smaller (20 inches long by $1\frac{1}{4}$ inches square section), is heated to a known temperature and its rate of cooling observed, the object being to determine the rate at which any element of surface of the statical bar loses heat.

From the results of these two experiments, together with the specific volume (the reciprocal of the density) and the specific heat of the iron, the flow of heat across any portion of the statical bar may be evaluated, thus giving the heat conductivity.

Forbes's method postulates that the material under examination suffer no marked change in structure during the experiment. Consequently this method is applicable to the determination of the heat conductivity of soils and powders only by radical modification, since it is impossible to obtain a powder twice in the same condition as to packing, moisture content, flocculation, etc.

This modification of Forbes's method consists in measuring simultaneously the *rate of temperature rise* in a soil at regular distances

^a Cf. Preston, *Theory of Heat*, ed. 1904, pp. 632-633.

^b Phil. Trans. Roy. Soc. Edinburgh, **23**, 133, April, 1862. See also Preston, *Theory of Heat*, 1904, p. 640, from which this reference is cited.

from the constant temperature heat source, and the *difference of temperature gradient* which causes this rise.

Consider an insulated box of dry soil one end of which is maintained at a constant temperature of 100°C ., uniform over the cross section, figure 2, and the other end maintained at a lower temperature, say 0°C . Then, after a long period the temperature along the box will be proportional to the distance from the heated end. If the box is exactly 100 cm. long, the temperature will decrease 1 degree for every centimeter, when the soil temperature at every point has ceased to change.

Before the box of soil attains this steady state the temperature at each point in the soil is changing and the fall of the temperature along the box at any instant will be represented by some such curve as curve 2, figure 2. In this condition the heat which flows in at one surface of the element A is greater than that which flows out at the

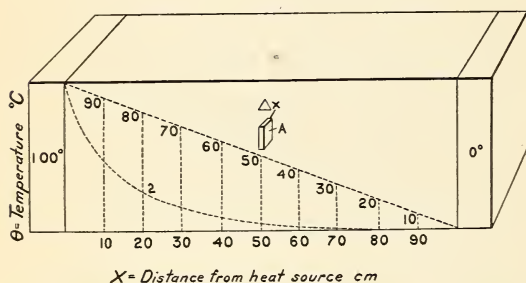


FIG. 2.—Temperature gradients in steady state and in unsteady state.

surface farther removed from the heat source. If the mean temperature of the element rises by an amount $d\theta$ in a time dt , a quantity of heat, $Ac \left(\frac{d\theta}{dt} \right) \Delta x$ is spent per second in raising the temperature of the soil, where c is the thermal capacity of

the soil—that is, its effective specific heat divided by its apparent specific volume— A is the area of the element, and Δx its thickness. The excess of heat entering the element over the heat which flows out at the farther surface of element $A\Delta x$, is now made up of two terms, one, due to surface radiation over the surface presented by the edge Δx with perimeter, p ; the other, due to heat used in raising the temperature of the soil element, $A\Delta x$. Thus,

$$AK \left(\frac{d^2\theta}{dx^2} \right) \Delta x = Ep \Delta x \theta + Ac \left(\frac{d\theta}{dt} \right) \Delta x.$$

If the soil box be insulated, the emissivity (E) may be taken as zero and the equation becomes

$$\frac{K}{c} \left(\frac{d^2\theta}{dx^2} \right) = \frac{d\theta}{dt}$$

or $K = \left(\frac{c}{\frac{d\theta}{dt}} \right) c \div \left(\frac{d^2\theta}{dx^2} \right).$

Consequently, to determine the absolute heat conductivity of a soil, it is necessary to know $\left(\frac{d\theta}{dt}\right)$, $\left(\frac{d^2\theta}{dx^2}\right)$, and c . The diffusivity is given by dividing $\left(\frac{d\theta}{dt}\right)$ by $\frac{d^2\theta}{dx^2}$.

The rate $\left(\frac{d\theta}{dt}\right)$ at which the temperature rises with time at a known distance x from the heat source is read on a thermometer and with a stop watch.

$\frac{d^2\theta}{dx^2}$ is the rate at which the fall of temperature along the box decreases on passing from the heated end. This is found by three successive steps: (a) The rate of rise with time is read on each of a series

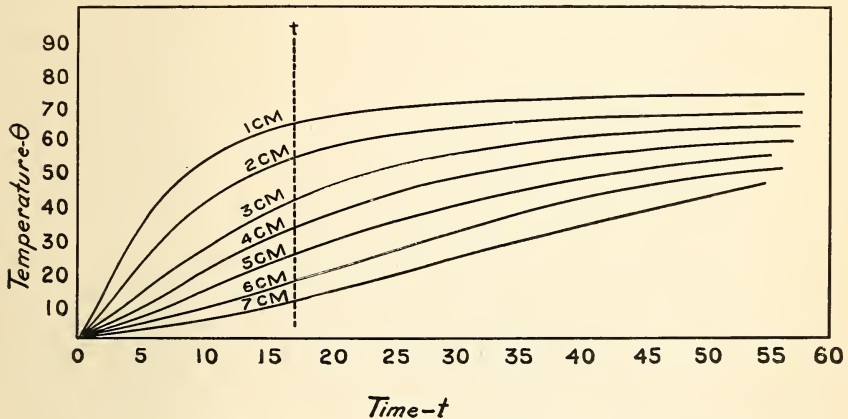


FIG. 3.—Temperature-time curves for the general case.

of thermometers placed in the central axis at known distances from the heated end of the soil box. These readings are then plotted, using temperature as ordinates and time as abscissas. Such a series of curves is shown in figure 3. (b) At the particular instant, t , when a thermometer (say No. 5 thermometer at a distance of 5 cm. from the heat source) shows a rate of rise of $\left(\frac{d\theta}{dt}\right)$ (which may be from 0.3° to 1.0° per minute), a cross section of the curve in figure 3 is taken. This means that the reading of each thermometer at the instant t is taken from its curve in figure 3. With these values, we can construct a curve showing the fall of temperature along the soil box at the instant t , i. e., the temperature gradient, as shown in figure 4. (c) To calculate $\left(\frac{d^2\theta}{dx^2}\right)$, the rate of change of the temperature gradient at 5 cm. from the heat source, the gradient $\left(\frac{d\theta}{dx}\right)$ is taken from the curve

in figure 4 at 4.5 cm. from the heat source, and again at 5.5 cm. from the heat source. The difference $\left(\frac{\Delta\theta_{4.5}}{\Delta x_{4.5}} - \frac{\Delta\theta_{5.5}}{\Delta x_{5.5}}\right)$ divided by Δx (which here is 1 cm.) gives $\frac{\Delta^2\theta}{\Delta x^2}$ at 5 cm.

The value of c , the thermal capacity of the soil, is found by dividing the effective specific heat of the soil by its apparent specific volume.

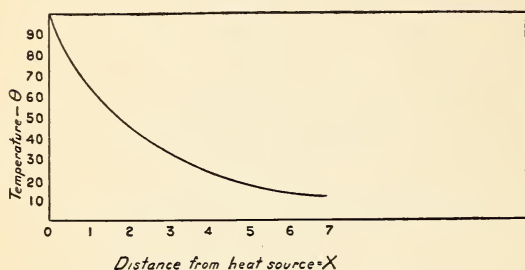


FIG. 4.—Temperature gradient for unsteady state.

The specific heat of a moist soil is obtained by adding the thermal capacity of the solid portion of the soil to the thermal capacity of the water, the contribution of the gases to the heat capacity of the soil being very small. The apparent specific volume is determined by weighing a known volume of the moist soil in the condition of packing in which it enters into the experiment, where its conductivity is to be determined.

In the foregoing description of the method used in determining heat conductivity, it is assumed, for the sake of simplicity, that the soil moisture remains evenly distributed throughout the box during the determination. In practice, however, the soil moisture passes from regions of higher to regions of lower temperature, thus causing the heat conductivity to vary (1) with the *time* at any one soil element, and (2) with the *distance* from the heat source, the dryer portions of soil near the heat source conducting less well than the moist soil farther out. The effect of this drying out of soil upon the temperature gradient is shown in figure 5.

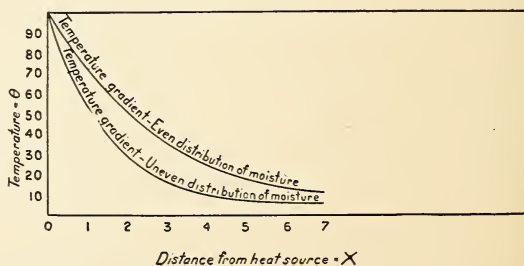


FIG. 5.—Effect of uneven distribution of moisture upon temperature gradient in unsteady state.

Curve 1 illustrates the fall in temperature along the box, supposing the soil moisture remains evenly distributed; curve 2 shows the effect of the drying out near the heat source upon the fall of temperature along the box; the poorly conducting dry layers of soil near the heat source require a high temperature gradient to force the heat through

them, consequently the temperature drop along the box is at first rapid and then very gradual.

It was to avoid this very drying out effect that the above-described method was chosen, consequently, the *rate of temperature rise with time*, $\left(\frac{d\theta}{dt}\right)$, and *rate of change in temperature gradient* $\left(\frac{d^2\theta}{dx^2}\right)$, are taken for a point in the soil so far removed from the heat source that no great disturbances in the moisture content or packing have yet taken place. It is this instantaneous reading which gives validity to this method, since any long-continued heating will change completely the condition and the heat conductivity of the soil.

APPARATUS.

The apparatus used in this method consists in the main of three distinct features: (1) The heat source; (2) the soil box; and (3) the mechanical packing apparatus.

(1) The heat source is a heavy rectangular brass tank, 17 cm. wide, 12 cm. deep, and 20 cm. long, insulated with asbestos and felt on five sides, one end being exposed to serve as a heat-contact. The tank is maintained at a constant temperature, near 100°C. , by boiling water in it

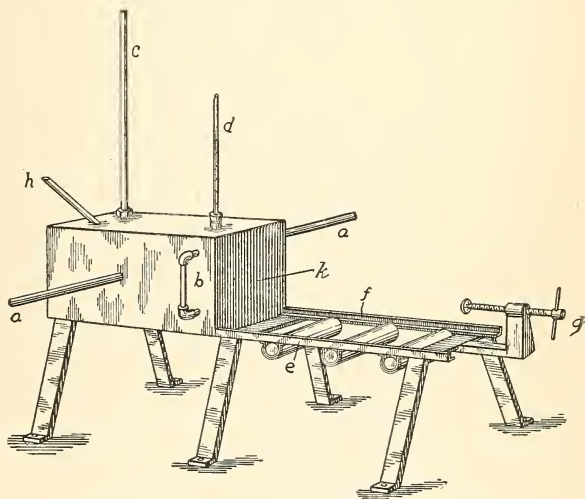


FIG. 6.—Heat source used in this investigation.

and allowing the steam to escape (*h*, fig. 6) at atmospheric pressure. For lower and higher temperatures, different liquids may be used. The liquid in the tank is very conveniently heated by a copper coil carrying live steam (*a*,) which passes horizontally through the tank. The steam exhausts into a drainpipe where it is condensed by running water. The level of the water in the tank is shown by the gauge *b*, and as the water boils away, the supply is replenished by the feed-pipe, *c*. The temperature of the tank is given by the thermometer, *d*; *e* and *f* are skids for the soil box, and *g* is a screw clamp serving to press the soil box firmly in place against the heat source.

(2) The soil box is built of pine wood boiled in paraffin and painted with asphaltum. One end is made of copper. Externally it has a coating of asbestos board cemented to the wood. The end cross section inside the box is 10 by 10 cm. Centigrade thermometers, reading to 110° , are inserted through rubber stoppers in the sides of this box at intervals of 1 cm., the even positions alternating with the odd positions on opposite sides of the box, thus, numbers

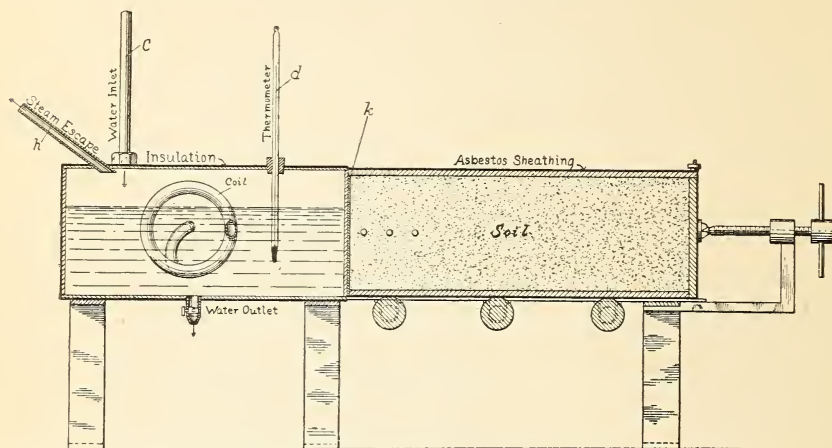


FIG. 7.—Longitudinal section of heat source with soil box in position.

1, 3, 5, 7 are on the right side of the box, and numbers, 2, 4, 6 on the left side (fig. 8). The thermometer bulbs are ranged along the central diameter of the box, which is perpendicular to the plane of the heat source. For some of the determinations thermometers were placed above and below this central diameter to ascertain the effect, upon the rate of the temperature rise with time, of departure from this central diameter.

In making a determination the soil box is fitted with the thermometers and then filled with soil of definite moisture content from the mechanical packing device to be described below. The soil box is then allowed to stand until the thermometers are nearly in a steady state at room temperature and the reading of each thermometer at "zero" time taken. The copper end of the soil box (*K*, figs. 6 and 7) is then placed against the heat source, a stop watch pressed at the instant of contact, and readings of time and temperature taken from each thermometer

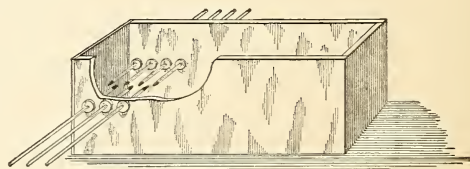


FIG. 8.—Soil box, showing arrangement of thermometers.

during a period which may cover twenty minutes or two hours, depending upon the heat conductivity of the soil.

(3) The mechanical packing apparatus used has been pictured and described in a previous bulletin^a and is reproduced in figure 9. It consists of an electric motor whose shaft bears an eccentric collar *E* fitting into a hole in the end of a sieve *S*. When the motor is going the sieve vibrates rapidly at its lower end, the upper end being held in place by wire hooks

R which engage a support and are kept in place by the pendant weight *W*. The tin false bottom *T* feeds the soil to the screen, which had three meshes to the centimeter in the work here presented. The soil box (fig. 8) may thus be filled with soil under definite mechanical conditions. In all the experimental

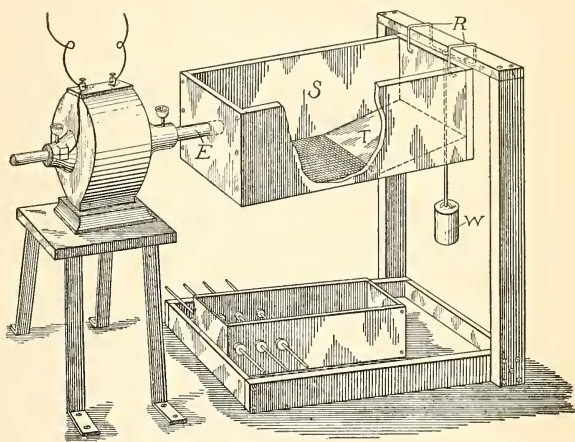


FIG. 9.—Mechanical packer, showing soil box below.

determinations of heat conduction the soil was packed by means of this device, the height of the sieve above the soil box being always the same.

GENERAL SURVEY OF EXPERIMENTAL WORK.

In order to ascertain the effect of water upon the heat conductivity of a powder whose grains are all of the same material, experiments were made using a coarse grade of crushed quartz at seven different moisture contents. Similarly a fine quartz flour was used at ten different moisture contents. With these two series of experiments in hand giving the change of heat conductivity and diffusivity produced in a single-substance powder by addition of various quantities of water; we are in a position to try the similar effect of moisture in more complex powders, such as soils.

A sand from the bottom of the Hudson River at five different moisture contents was then studied; next, Podunk fine sandy loam at eight moisture contents; Leonardtown silt loam at ten moisture

^a Moisture Content and Physical Condition of Soils. Cameron and Gallagher, Bulletin No. 50, Bureau of Soils, U. S. Dept. Agr., 1908, p. 60.

contents; and a muck soil containing 25 per cent of organic matter at eight moisture contents. Also Hagerstown loam, Galveston clay, and a No. 4 sand, a separate from a mechanical analysis, were studied in less detail.

With a different spacing of thermometers from that given above (where they were placed 1 centimeter apart) Norfolk sand in air-dry condition was also used. Finally, to determine how the heat conductivity of an air-dry powder varies with the size of grain, under the same conditions as to packing, a series of experiments with carborundum powder was carried out.

EXPERIMENTAL RESULTS.

COARSE QUARTZ.

The sample of quartz powder used was largely medium sand, as shown by the following mechanical analysis, which is the mean of several separate analyses:

TABLE I.—*Mechanical analysis of coarse quartz powder.*

Diameter of grain.	Conventional name.	Per cent.
<i>Millimeters.</i>		
2 - 1	Fine gravel.....	0
1 - 0.5	Coarse sand.....	14.74
0.5 - .25	Medium sand.....	58.22
.25 - .1	Fine sand.....	26.11
.1 - .05	Very fine sand.....	.61
.05 - .005	Silt.....	.00
.005- .000	Clay.....	.00

The basal observations of the temperature at distances of 1, 2, 3, 4, 5, 6, and 7 cm. from the heat source at various intervals of time are given in Table II. Six similar sets of observations were made for the same quartz flour at six other moisture contents. The experimental data are not presented at this time, but the tables are on file in the Bureau of Soils and are available to the scientific public.

TABLE II.—*Fundamental data for determination of heat conductivity and diffusivity—Con.*

RATE OF RISE OF TEMPERATURE IN SOIL AT DIFFERENT DISTANCES FROM A STEADY SOURCE OF HEAT—Continued.

5 cm. from heat source.		(Beckmann.)		6 cm. from heat source.		7 cm. from heat source.	
Time.	Temp.	Time.	Temp.	Time.	Temp.	Time.	Temp.
<i>m. s.</i>	<i>° C.</i>	<i>m. s.</i>	<i>° C.</i>	<i>m. s.</i>	<i>° C.</i>	<i>m. s.</i>	<i>° C.</i>
0 20	19.45	0 00	0.715	0 50	19.3	0 35	19.5
1 50	19.45	1 00	.715	3 00	19.3	7 10	19.5
4 20	19.45	3 00	.715	4 30	19.3	10 00	19.5
5 45	19.45	3 45	.715	7 45	19.3	15 40	19.65
7 00	19.45	4 55	.715	8 20	19.35	19 35	19.8
8 00	19.45	6 20	.715	9 40	19.35	28 35	21.0
8 55	19.45	7 25	.715	10 40	19.4	29 45	21.1
10 00	19.5	8 15	.715	11 10	19.4	34 45	22.3
10 50	19.55	9 00	.715	14 25	19.45	41 50	24.0
11 50	19.65	10 15	.745	15 10	19.55	45 00	25.0
12 55	19.7	11 00	.775	16 30	19.6	47 10	25.5
15 35	20.0	12 00	.835	17 25	19.7	50 40	26.5
16 10	20.05	12 50	.900	18 20	19.8	56 00	27.9
17 00	20.3	13 30	.975	19 15	19.95	58 45	28.7
17 45	20.50	14 43	1.145	20 35	20.2	60 30	29.0
18 00	20.6	15 03	1.200	21 15	20.3	62 45	29.6
19 35	21.0	15 53	1.350	22 20	20.6	65 10	30.1
20 55	21.5	16 37	1.500	23 05	20.7	67 10	30.7
22 55	22.2	17 15	1.650	24 35	21.1	69 30	31.2
23 30	22.5	17 51	1.800	25 25	21.3	72 25	32.0
24 28	22.9	18 15	1.900	26 15	21.6	74 00	32.3
25 34	23.3	19 07	2.150	26 55	21.7
26 35	23.8	19 48	2.350	28 05	22.1
27 12	24.0	19 58	2.400	31 45	23.3
27 55	24.4	20 07	2.450	33 30	24.0
30 00	25.4	20 16	2.500	35 45	24.8
31 00	25.9	20 26	2.550	37 00	25.3
32 00	26.3	21 02	2.750	39 00	26.0
33 00	26.8	21 20	2.850	40 45	26.7
34 00	27.2	21 30	2.900	42 00	27.2
35 00	27.7	22 03	3.100	43 20	27.7
36 15	28.1	23 00	3.450	44 40	28.0
37 15	28.6	23 33	3.650	46 55	28.9
38 30	29.2	23 41	3.700	48 00	29.3
40 15	30.0	23 49	3.750	49 20	29.8
41 25	30.5	23 57	3.800	50 30	30.1
42 15	30.85	24 03	3.850	52 00	30.6
43 00	31.1	24 12	3.900	53 15	31.0
44 00	31.5	24 43	4.100	54 35	31.4
45 30	32.1	24 58	4.200	56 10	32.0
46 30	32.5	25 05	4.250	59 00	32.8
47 30	33.0	25 44	4.500	60 00	33.5
48 40	33.3	25 59	4.600	63 00	34.0
50 15	33.9	26 22	4.750	64 55	34.6
51 30	34.3	26 43	4.900	66 45	35.0
53 00	34.9	26 57	5.000	69 15	35.7
54 10	35.2	27 17	5.150	73 40	36.8
55 10	35.5	27 26	5.200
56 20	35.9	27 33	5.250
57 50	36.3
59 10	36.8
60 15	37.0
61 45	37.5
64 50	38.3
66 00	38.6
67 30	39.0
68 45	39.3
71 30	40.0
74 15	40.6

Temperature of heat source, 98.8.

Heat applied at 3 minutes 30 seconds.

Apparent specific volume 0.601.

For Table II the quartz powder was air dry. For the remaining six determinations the quartz was moistened and thoroughly well mixed by hand and by several passages through the screen of the mechanical packer (fig. 9). It was then allowed to fall into the soil

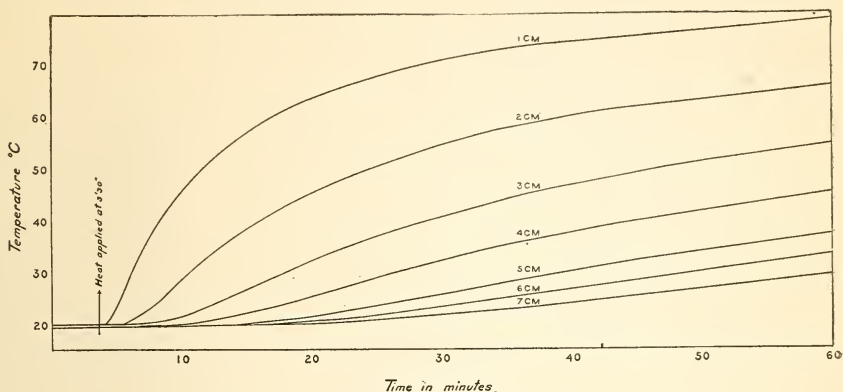


FIG. 10.—Temperature-time curves for air-dry coarse quartz powder.

box (fig. 8) from the mechanical packer and a volume weight taken for part of the same sample by allowing it to pack from the screen of the packer into a glass crystallizing dish of known volume, which was weighed after scraping the top of the quartz powder flush with the edge of the dish. The moisture content of this sample in the crystallizing dish was then obtained by evaporating first on a steam bath and finally to constant weight in a toluene oven at 110° C. The thermometers in the soil box were read, and if their temperatures were close together and near the room temperature, the box was applied at once to the source of heat and the time-temperature readings begun. In some experiments the thermometers were read for some time before applying the box to the source of heat.

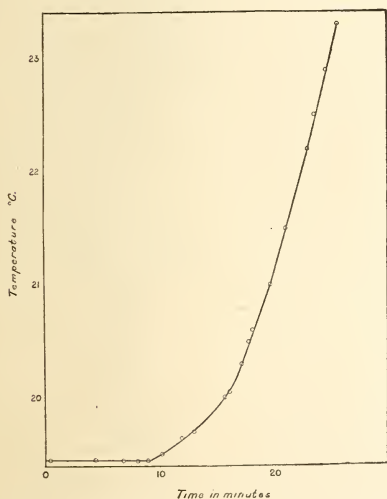


FIG. 11.—Temperature-time curve for air-dry coarse quartz powder, No. 5 thermometer, showing period of straight line rise.

In some of the experiments a Beckmann differential thermometer was inserted at a distance of 5 centimeters from the source of heat in order to read the temperature differences more accurately at this point. The thermometers used for the general work were graduated to 1° C., and could be read by means of a hand lens to 0.1°. Their

range was -10° to $+110^{\circ}$ C. The bulb of each thermometer was about 2.7 cm. long and 0.5 cm. in diameter. The time-temperature curves were plotted with temperature as ordinates and time as abscissas, the readings for all seven thermometers at one moisture content appearing on the same sheet, figure 10. With these curves as a basis the fall in temperature along the soil box may be interpolated for *any instant* and these temperature values plotted as a function of the distance x of each thermometer from the source of heat, figure 12. The particular instant at which this gradient curve, figure 12, is to be taken is found by plotting on a larger scale than figure 10 the time-temperature curve for the thermometer 5 cm. distant from the

heat source, as shown in figure 11. Here it is seen that after a short period the temperature rises at a nearly constant rate, -0.433° C. per minute in this particular experiment. The instant at which this constant rate is established is taken as the time at which to read the temperature gradient from the fundamental curve given in figure 10. From this temperature gradient, as plotted in figure 12, a value of the rate of change of temperature gradient with distance from the

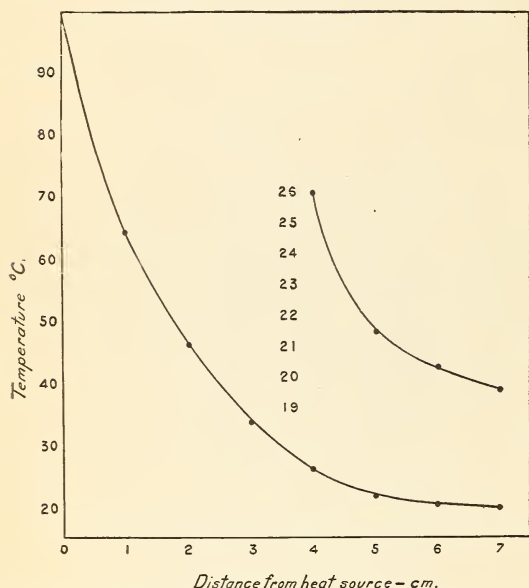


FIG. 12.—Temperature gradient for air-dry coarse quartz powder plotted to two different scales.

heat source at the particular instant, t , may be found, i. e., $\frac{d^2\theta}{dx^2}$, either graphically or analytically. These values, $\frac{d\theta}{dt}$ and $\frac{d^2\theta}{dx^2}$, together with the apparent specific volume and the effective specific heat, are assembled in Table III, and from them the absolute heat conductivity and diffusivity are calculated and given in the columns so designated. The curves for time-temperature readings and for temperature fall along the soil box are not given for each experiment, since such a full treatment would render this paper unnecessarily bulky, and since such curves to be of practical benefit need to be given with an accuracy which can be best attained by plotting each set of curves from such data as that given in Table II to a larger scale than published curves

of this sort will admit. This method of treatment as shown in figures 10, 11, and 12 will be adhered to throughout this paper, consequently one complete analysis of the method used in calculating $\frac{d\theta}{dt}$, $\frac{d^2\theta}{dx^2}$, and the values for the absolute heat conductivity and diffusivity derived therefrom will suffice. In Table III there are also given a number of values for $\frac{d\theta}{dt}$ at 5 cm. from the heat source and under known conditions of packing and moisture content, but the auxiliary thermometers were not used to secure the temperature gradients in these experiments, so that the value $\frac{d^2\theta}{dx^2}$ is unknown. These values, however, serve as a check on the value $\frac{d\theta}{dt}$, and on the specific volume for the moisture contents given.

TABLE III.—Summary of experimental and derived data.

COARSE QUARTZ.

[illegible]

FINE QUARTZ FLOUR.

[illegible]

HUDSON RIVER SAND.

	9.84	10.92	.2697	.761	23.0	9 25	15 35	2.00	.2052	2127	.00354	.00177	.380
.965	9.84	10.92	.2697	.761	23.0	9 25	15 35	2.00	.2052	2127	.00354	.00177	.380
1.057	10.83	12.15	.2777										
1.080	14.31	16.70	.3059										
.972	16.42	19.67	.3230	.887	27.5	10 25	14 35	1.80	.2865	.3050	.00508	.00282	.493
.549	21.04	26.65	.3604	.625	27.0	9 25	16 10	1.28	.2452	.4100	.00683	.00534	.488

PODUNK FINE SANDY LOAM.

	21 10	20 10	25 35	2.00	.0063	0.0950	0.001582	0.000792	0.179
0.698	0.257	0.298	1.33	2.36	.0985	.1215	.00203	.000792	.198
.840	2.11	2.14	2.00	2.80	.1397	.1453	.00243	.000867	.198
.926	2.76	2.83	.2653	2.40	.1457	.1457	.00243	.001012	.296
1.000	6.19	6.601	.2534	2.88	.2552	.2165	.00361	.00153	.350
1.085	9.15	10.08	.2575	2.40	.2575	.2495	.00417	.001735	.417
1.032	16.84	20.25	.3204	2.40	.3185	.3710	.00619	.00258	.414
.859	21.22	26.93	.3562	1.00	.1727	.3290	.00548	.00548	

LEONARDTOWN SILT LOAM.

	20 45	15 35	22 30	2.4	0.1056	0.1275	0.00213	0.000886	0.1850
0.828	0.800	0.806	2.127	3.0	.1375	.1567	.00261	.000870	.1806
.878	2.083	2.127	2.455	2.2	.1392	.2305	.00385	.001750	.3532
.865	3.450	3.580	.2563	2.8	.2008	.2090	.00348	.001243	.2718
.901	4.470	4.690	.2574	2.4	.2440	.2642	.00417	.001837	.3470
.924	8.240	8.980	.2829	2.2	.2725	.3070	.00513	.002332	.408
.878	9.620	10.650	.3035	2.8	.2760	.2702	.00451	.001610	.3214
1.002	10.367	11.570	.3065	2.6	.3287	.3231	.00538	.002070	.3646
1.017	15.150	18.350	.3465						
.508	26.370	35.810							
.561	26.240	35.580							

^a Specific heat of dry soil taken as 0.1900.^b Saturated with water.

TABLE III.—Summary of experimental and derived data—Continued.
MUCK SOIL.

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)
Apparent specific volume, f	Moisture content on wet weight	Moisture content on dry weight.	Effective specific heat C_a	Rate of rise of temperature at 5 cm. from heat source (No. 5 thermometer, $\frac{d\theta}{dt}$)	Temperature at foot of No. 5 thermometer for straight line rise.	Time at foot of No. 5 thermometer straight line rise.	Period required for rise of 6.3° C. at 5 cm. from heat source.	Rate of change of temperature gradient at 5 cm. from heat source for $\frac{d\theta}{dt} = 0.5 \text{ cm.} \frac{d^2\theta}{dx^2}$	$(4) \times (5) =$ calories per gram of soil per minute	$(10) \div (1) =$ calories per cubic centimeter per minute	$(11) \div (60) =$ calories per cubic centimeter per second.	$(12) \div (9) =$ calories per square centimeter per degree change in temperature gradient $= K \frac{C(d\theta)}{dt} \div \left(\frac{d^2\theta}{dx^2}\right)$	$(5) \div (9) =$ diffusivity rate of rise of temperature at 5 cm. from heat source per degree change in temperature gradient, $\frac{C(d\theta)}{dt} \div \left(\frac{d^2\theta}{dx^2}\right)$
C_r	Per cent.	Per cent.	Calories.	$^{\circ}\text{C. per minute}$	$^{\circ}\text{C.}$	Min. sec.	Min. sec.	$^{\circ}\text{C. per cm.}$					$^{\circ}\text{C. per minute.}$
0.805	3.78	3.35	0.1884	0.427	26.3	28 30	31 30	4.28	0.0804	0.0898	0.001495	0.000319	0.0099
1.250	18.67	22.35	.3140	.510	24.0	30 00	34 10	2.80	.1022	.1282	.002138	.000763	0.0599
1.372	32.0	47.00	.4265	.530	24.0	29 40	35 25	3.20	.2240	.1137	.002335	.000718	.1521
1.403	37.1	58.98	.4605	.485	28.9	21 00	27 20	2.40	.2275	.1530	.002595	.001081	.202
1.488	38.6	62.83	.4822	.516	29.7	20 15	25 40	2.60	.2475	.1673	.00277	.001066	.198
1.407	40.97	69.42	.5021	.535	32.7	27 55	35 15	2.72	.1984	.1110	.00235	.000864	.145
a 54.3	119.2	119.2	.6145	.292	23.0	23 00	38 18	1.72	.1733	.2220	.00370	.00215	.170
.765	a 55.46	123.0	.6218	.274	23.0	27 00	42 25	2.24	.1704	.2230	.00371	.00166	.122
HAGERSTOWN LOAM.													
0.949	1.11	1.12	0.2004	0.397	29.0	19 35	28 50	2.00	0.796	0.0838	0.00140	0.000699	0.199
.602	32.45	48.06	.4538	.390	26.9	10 09	18 00	2.00	.268	.445	.00741	.00370	.255
GALVESTON CLAY.													
0.949	1.39	1.41	0.2207	0.388	29.2	24 40	34 00	2.6	0.0836	0.0902	0.00150	0.000577	0.192
.654	40.31	67.55	.5283	.433	25.6	13 30	24 25	1.6	.2287	.3495	.00576	.00360	.2706
NO. 4 SAND.													
0.648	0.018	0.0198	0.1836	0.382	25.4	21 50	31 40	2.4	0.07015	0.1080	0.0018	0.00075	0.1591

Specific heat of dry Hagerstown soil 0.1914
 Galveston soil2097
 No. 4 sand1835
 a Flows.

In Table III several columns of auxiliary data are given in addition to that required merely for the calculation of the heat conductivity and diffusivity. Column 1 contains the experimental value for the apparent specific volume at the moisture content given in column 2, while column 3 shows this same moisture content calculated to percentage on the dry weight of the soil, and column 4 gives the effective specific heat calculated from the moisture content in column 3, the specific heat of solid quartz being taken as 0.1900 and that of water as 1. Column 5 shows the rate of rise of the thermometer at 5 cm. from the heat source *per minute*; column 6 gives the temperature at which this rate of rise begins to be constant; and column 7 the *time*—from the application of the soil box to the source of heat—at which this rate of temperature rise sets in. Column 8 gives the period of time required for the temperature of the soil at 5 cm. from the heat source to rise 6.3° C., an interval of temperature chosen arbitrarily, but serving to compare the effect of moisture in soil upon the rate at which the soil heats up at a fixed distance from a constant-temperature heat source whose heat supply is infinite. Column 9 contains the graphically calculated values of $\frac{d^2\theta}{dx^2}$, obtained in each case by use of three sets of curves as shown in figures 10, 11, and 12, and given here for $\Delta x = 0.5$ cm. in every case. Column 10 shows the product of the effective specific heat (column 4) and the rate of temperature rise per minute; this product in column 10 is divided by the apparent specific volume (column 1) and the result given in column 11, which again is divided by 60, to reduce the time unit from minutes to seconds, and the result placed in column 12. This last column again is divided by $\frac{d^2\theta}{dx^2}$ as given in column 9 and the final value for K , the absolute heat conductivity obtained, as shown in column 13. Column 14 contains the diffusivity or rate at which the temperature rises under a unit temperature gradient. This is calculated by dividing the rate of rise of temperature at a distance of 5 cm. from the heat source by the rate of change of temperature gradient with distance from heat source, i. e. $\left(\frac{d\theta}{dt}\right) \div \frac{d^2\theta}{dx^2}$.

In figure 13 the apparent specific volume, the heat conductivity, and the diffusivity have been plotted as ordinates, and the moisture contents of the quartz as abscissas. The special scale for ordinates used for each curve is shown at the left for the heat conductivity and at the right for the diffusivity and specific volume. The specific volume curve shows an optimum water content at a point somewhere between 2 and 4 per cent of moisture on the wet weight. The diffusivity curve goes through a maximum value, but the maximum shows no direct relation to the optimum water content. This maximum

diffusivity is due to the fact that a little water in the quartz affords contact between the grains and breaks down the air insulation, but as more and more water is added the heat capacity of the moist quartz is raised, and in consequence thereof the temperature rise, produced by a given quantity of heat, is necessarily lowered. The curve for heat conductivity rises regularly, and at the moisture contents studied approaches a maximum value. The last point taken (20.26 per cent



FIG. 13.—Effect of moisture upon the apparent specific volume, heat conductivity, and diffusivity of coarse quartz powder.

of water on wet weight) was for quartz saturated with water; at higher moisture contents the heat conductivity would gradually fall to the heat conductivity of water. Consider now the time required for the temperature at a distance of 5 cm. from the heat source to rise 6.3° C., as shown in column 8. For the air-dry quartz it varies from 35 minutes to 27 minutes 20 seconds, depending on the moisture

content and packing. Addition of 1.2 per cent of moisture reduces this time to 17 minutes 40 seconds, some 50 per cent; but further addition of water only slightly decreases this time, which is again increased as the quartz becomes saturated with water. From these last observations it is evident that the determination of the time required by any *one point* in the soil to rise through a stated temperature interval can not be taken as a measure of the ability of the soil to conduct heat.

SOURCES OF ERROR.

Effect of working in the "unsteady state."—The necessity for taking temperature-time curves in the unsteady state, to avoid evaporation of soil moisture from the layer in which it is desired to measure the heat conductivity, introduces the factor $\frac{d^2\theta}{dx^2}$, the acceleration of the temperature with respect to distance from the heat source. And the evaluation of this factor $\frac{d^2\theta}{dx^2}$ requires the use of second differences for temperature data, which data are of such a nature that these second differences may lie very close in value to the error of experiment.

Two methods of calculating $\frac{d^2\theta}{dx^2}$ are available: (a) The graphical method, which has been described above, consists in measuring off a value of $\Delta\theta$ and of Δx on the temperature-distance curve given in figure 12 and taking this ratio $\frac{\Delta\theta}{\Delta x}$. If we do this at two points on the curve at a distance Δx from each other and then subtract these two ratios $\frac{\Delta\theta_1}{\Delta x_1}$ and $\frac{\Delta\theta_2}{\Delta x_2}$, and divide the remainder by Δx , we have performed the operation indicated by the symbol $\frac{d^2\theta}{dx^2}$ and get its value at once. The difficulty is that different magnitudes of Δx give different values of $\frac{d^2\theta}{dx^2}$, and the choice of the proper Δx must be made so that it is considerably greater than the experimental error and still is not so large as to cover too great a length of the curve, θ, x , whose direction is rapidly changing. In this paper Δx has been taken as 0.5 cm. and the various values of $\frac{d^2\theta}{dx^2}$ worked out upon that basis.

(b) The analytical method of evaluating $\frac{d^2\theta}{dx^2}$ consists in determining the empirical constants of the curve, θ, x , from the observed data.

The advantage in this method lies in that it enables us to use rigid differentiation, the $\Delta\theta$ and Δx being carried to the limit and becoming $d\theta$ and dx , infinitesimally small. The disadvantage of the method is

the immense quantity of detailed computation involved. On this account we have used the less exact graphical method of deriving $\frac{d^2\theta}{dx^2}$. Later, if it is thought that the other experimental errors which limit the accuracy of this work warrant it, this analytical method of evaluating $\frac{d^2\theta}{dx^2}$ may be taken up.

Spacing of thermometers.—The thermometers were placed with the central diameters 1 cm. apart. A deviation of 0.05 mm. from the correct position would mean an error of 5 per cent. Consequently the values given throughout this paper may be 5 per cent high or low from this source of error alone.

Dimensions of thermometers.—The diameters of the thermometer bulbs varied as much as 1 mm. from the average value 5 mm. This error might add to or subtract from the error of placement (No. 2), depending upon the signs (+ or -) of each error involved.

Reading of thermometers.—The error due to slow response of the mercury thread in the thermometer to change in temperature is a much more serious error factor than the personal error involved in taking the position of the mercury thread with a hand lens. The position of the mercury meniscus may be determined certainly within a fifth of a degree and very nearly to a tenth with the thermometers used. This slow response of the mercury to temperature change was partially overcome by tapping each thermometer before reading it: but where temperature-time readings are taken, the general trend of the curve is all that can be hoped for, even using a Beckmann differential thermometer graduated to 0.01° C. and a 0.2 second stop watch.

Reading of stop watch.—The time was read to seconds on the stop watch, but since the same observer reads the mercury of the thermometer and the watch, there is a lack of coordination between temperature and time in the readings which amounts to an error of from one-half to one second. This error together with the sticking of the mercury thread prevents the use of thermometers of a fine graduation.

Determination of moisture.—The moisture determinations were made using samples of the soil from the *same batch* as that used to fill the soil box; the samples were taken at the time when the soil box was being filled from the mechanical packer, and consequently were in the same state as the soil in the soil box. In several cases the soil remaining in the box after a heat conductivity experiment had been finished was tested and its moisture content found to be nearly the same as at first *in the part of the soil beyond the thermometer 5 cm. from the heat source*. Distillation of moisture of course takes

place near the heat source, but this does not extend in a marked way much beyond 3 cm. out from the heat source. The lack of an accurate soil hygrometer prevents exact determination of this distillation phenomenon. Finally it may be stated that the moisture-content value is the least subject to error of any of the factors in this research. The experimental errors involved in its determination are so slight as to have no appreciable influence upon the effective specific heat of the soil as calculated from the moisture content.

Apparent specific volume.—This factor in the problem is very difficult to control. The packing of the soil at the bottom of the soil box is closer than at the top, owing to the difference in height of fall from the sieve of the mechanical packer (fig. 9) and to the weight of the soil. This effect is obvious from theoretical considerations, and was experimentally demonstrated by volume-weight determinations made under the conditions of this experimental investigation. Consequently the apparent specific volume was determined as an average value from top to bottom of the soil box by allowing the soil, in the same condition as to moisture as that in the box, to fall from the mechanical packer into a deep, wide crystallizing dish sufficiently large to represent the volume conditions obtaining in the soil box. A number of duplicate volume-weight determinations showed the precision of this method so far as an individual batch of soil was concerned. It can not be too strongly emphasized, however, that the same soil with practically the same moisture content gives considerable variation in the apparent specific volume, owing to differences in the state of aggregation of the soil grains, which arise probably from otherwise undetected variations in the distribution of moisture. On this account, then, it is not surprising if duplicate determinations of the heat conductivity and diffusivity of soils show a considerable variation. However, the results obtained in this investigation show much greater regularity than might be expected.

The specific heat.—The determination of the specific heat of each soil in oven-dry condition was made by heating 100 grams of the sample to nearly 100° C. in a steam bath and then quickly stirring the hot soil into the water in a calorimeter.^a The rise in temperature was taken on a Beckmann differential thermometer graduated to 0.01° C. and the customary calculation made. A separate experiment, using the same quantities of water and of soil at the same temperature (about 25° C.), gave the temperature effect due to the simple wetting of the dry soil. This effect was allowed for in all the specific heat determinations made in this investigation.

Table IV shows the magnitude of this effect for several soils.

^a Compare Ulrich, Forsch. Geb. Agr. Phys. **17**, 1 (1894).

TABLE IV.—*The heat evolved by wetting certain soils, dried at 110° C.*

Weight of soil.	Water equivalent of calorimeter system.	Temperature rise.	Specific heat of dry soil (corrected).	Water equivalent of soil.	Total water equivalent of system.	Heat evolved by wetting soil.	Heat evolved per gram of soil.	Temperature of observation.	Name of soil.
<i>Grams.</i>	<i>Grams.</i>	<i>° C.</i>		<i>Grams.</i>	<i>Grams.</i>	<i>Calories.</i>	<i>Calories per gram.</i>	<i>° C.</i>	
100	515.3	0.070	0.1848	18.48	533.8	37.4	0.374	24	Norfolk sand.
100	515.3	0.060	18.48	32	.320	
100	511.87	1.360	.1566	15.66	527.53	716	7.16	24.2	
100	515.3	1.190	.1566	15.66	530.96	632	6.32	25.3	Muck soil, 25 per cent organic matter.
100	515.3	1.085	.1566	15.66	530.96	576	5.76	24.8	
100	532.87	.662	.2097	20.97	553.84	366	3.66	24.5	Galveston clay.
100	515.3	.730	.2097	20.97	536.27	391	3.91	25.2	Do.
100	515.3	.205	.1914	19.14	534.44	109.5	1.095	24.3	Hagerstown loam.
100	515.3	.210	.1914	19.14	534.44	112.2	1.122	24.4	Do.
100	515.3	.155	.1944	19.44	534.74	83	.83	24.3	Leonardtown silt loam.
100	515.3	.155	.1944	19.44	534.74	83	.83	26	
100	515.3	.050	.1828	18.28	533.58	26.7	.267	25	Podunk fine sandy loam.
100	515.3	.025	.1828	18.28	533.58	13.3	.133	24.5	Do.
100	515.3	.030	<i>a</i> .1900	19	534.3	16.02	.160	24.6	Large quartz grains.
100	515.3	.026	<i>a</i> .1900	19	534.3	13.88	.139	25.6	
100	515.3	.080	<i>a</i> .1900	19	534.3	42.7	.427	24.2	Fine quartz flour.
100	515.3	.057	.1900	19	534.3	34.5	.345	24.5	
100	515.3	.045	.1769	17.69	533	25	.250	23.5	Hudson River sand.
100	515.3	.020	.1769	17.69	533	10.7	.107	23.7	
100	515.3	.080	.1799	17.99	533.29	42.7	.427	24.5	No. 4 sand. Soil separate.
100	515.3	.077	.1799	17.99	533.29	41.1	.411	25.4	

a Assumed from data in Landolt & Börnstein.

The effect of size of soil grain is well shown by the two grades of quartz flour, the coarse evolved 0.160 and 0.139 calories per gram, while the fine quartz, which is nearly all a silt, evolved 0.427 and 0.345 calories per gram. The heat effect of wetting grades down from 7.16, 6.32, and 5.76 calories per gram for a muck soil containing 25 per cent of organic matter, to as low as 0.25 and 0.107 calories per gram for a Hudson River sand.^a The influence of this heat of wetting factor in specific heat determinations may be very great. Thus the ordinary muck soil gives an apparent specific heat of 0.2400, but when correction for the heat of wetting is made the true value of the specific heat is seen to be 0.1566, an error of over 33 per cent.

In calculating the effective specific heat of a soil at any one moisture content, one simply adds the heat capacity of the water to that of the dry soil.

Contact of soil box with source of heat.—There is no doubt that perfect contact between the copper end of the soil box and the brass end of the heat source was not secured. However, the temperature gradient and rate of rise of the soil thermometers furnish the necessary data for the purpose in hand, so that the exact temperature of the soil at 0.2 to 0.6 cm. from the heat source makes no difference in the result obtained for the heat conductivity and diffusivity, while it certainly does make a great difference in the quantity of heat which passes along into the soil box.

^a Compare Mitscherlich, Landw. Jahrb. 30, 361 (1901); 31, 577 (1902); also Jour. f. Landw. (1900), p. 71.

Variation in the temperature of the heat source.—The variations in the temperature of the heat source were not greater than 0.2° C. and usually were not of sufficient duration to show in the rate of rise of the soil thermometers. The error from this source is negligible.

Loss of heat by radiation.—The soil box was well insulated and the thermometers in the central diameter were each protected by some 4 cm. of soil from the influence of the sides of the box. Consequently the radiation effect may be neglected.

FINE QUARTZ.

The fundamental data for the temperature-time curves were taken, using a fine quartz flour at various moisture contents, the method of procedure being exactly as described above for the coarse quartz powder. Table V gives the mechanical analysis of this fine quartz flour, which is seen to be 80.5 per cent silt.

TABLE V.—*Mechanical analysis of fine quartz flour.*

Diameter of grain.	Conventional names.	Per cent. ^a
<i>Millimeters.</i>		
2-1.....	Fine gravel.....	0.00
1-0.5.....	Coarse sand.....	1.01
0.5-0.25.....	Medium sand.....	.09
.25-.1.....	Fine sand.....	.16
.1-.05.....	Very fine sand.....	8.00
.05-.005.....	Silt.....	80.47
.005-0.0.....	Clay.....	10.84
Total.....	99.57

^a Mean of ten samples.

The values of $\frac{d\theta}{dt}$ and $\frac{d^2\theta}{dx^2}$ were derived for each moisture content by the method described in detail for coarse quartz. These values, $\frac{d\theta}{dt}$ and $\frac{d^2\theta}{dx^2}$ are assembled in Table III (p. 26), together with the respective moisture contents and apparent specific volumes as determined experimentally for each of the experiments. In addition to the data summarized in Table III, a more extended series of experiments showing the dependence of apparent specific volume upon moisture content is given for fine quartz flour in Table VI.

The heat conductivity, K , the diffusivity, k , and the apparent specific volume, V , are each plotted in figure 14 as a function of the moisture content. The abscissas are per cents of moisture reckoned on the wet weight of quartz, and the ordinates are K , k , and V as shown by their respective scales in the right and left hand margins of figure 14.

From these three curves it appears that the heat conductivity rises in a fairly regular manner with increase in moisture content.

TABLE VI.—*Specific volume of fine quartz flour at various moisture contents.*

Moisture content on wet weight.	Specific volume.	Moisture content on wet weight.	Specific volume.
<i>Per cent.</i>	<i>C. c.</i>	<i>Per cent.</i>	<i>C. c.</i>
0.0833	1.130	9.84	0.965
.0833	1.151	10.22	1.110
.300	.927	10.4	.858
1.32	.947	10.83	1.057
2.97	.985	10.96	1.087
4.11	1.093	11.37	1.072
4.68	1.039	14.31	1.030
5.82	1.021	15.15	.911
6.48	1.040	16.42	.972
6.54	.992	17.78	.998
7.91	1.136	21.7	.683
8.02	1.008	22.68	.527
8.70	1.028	21.04	.549
8.80	1.050		

The influence upon the heat conductivity of the change in apparent specific volume is to lower the heat conductivity as the quartz assumes a more open structure in the neighborhood of the optimum water content (9 to 12 per cent on the wet weight). This relation is obscured by the difficulty experienced in securing uniform packing at each moisture content, so that the heat conductivity and diffusivity shown by lines which pass through actual experimental points are probably not so truly representative of the normal trend of the respective curves as the black-line curves drawn through the average of the points. It must be remembered, too, that error in the determination of $\frac{d^2\theta}{dx^2}$ in these places may be partly the cause of the de-

parture of the value of K or k from the average curve. The apparent specific volume curve shows that a very dry, fine powder occupies a greater volume than when very slightly moistened. And in this case the greatest volume and lowest heat conductivity and diffusivity are shown by the air-dried quartz, which held only 0.083 per cent of water on the wet weight.

Comparing the coarse quartz, figure 13, with the fine quartz, figure 14, it is evident that the dry, fine quartz conducts heat much more poorly than the dry, coarse quartz in the ratio of about 4 to 8.7, and the diffusivities are as 1.44 for fine quartz to 1.65 for coarse quartz. At higher moisture contents this relation is maintained, the coarse quartz increasing more rapidly in its heat conductivity as water is added than does the fine quartz. But as the saturation point of the powders for water is approached the heat conductivity of the fine quartz becomes greater than that of the coarse quartz until it is nearly twice its value.

The apparent specific volume of the coarse quartz lies below that of the fine quartz at all moisture contents.

The diffusivity of fine quartz shows only a slight tendency to pass through a maximum as moisture increases, whereas in the case of the coarse quartz this maximum is pronounced. It may be that with a greater quantity of water in the quartz this diffusivity will be less, and thus show a real maximum at 16 per cent of water, where now it is merely indicated. The last diffusivity measurement was made using quartz flour containing sufficient water to make a batter which poured very easily, so further experiments have to meet the difficulty that the quartz flour will settle from suspension during the measurements of heat conductivity and diffusivity, thus masking the true values.

HUDSON RIVER SAND.

The heat conductivity and diffusivity of a Hudson River sand^a were determined in the same manner as for the quartz powders. The values $\frac{d\theta}{dt}$ and $\frac{d^2\theta}{dx^2}$ together with the values of the apparent specific volume and moisture content are assembled in Table III (p. 27) for four different moisture contents, and in this same table are given the calculated heat conductivity and diffusivity. A mechanical analysis of this soil is shown in Table VII.

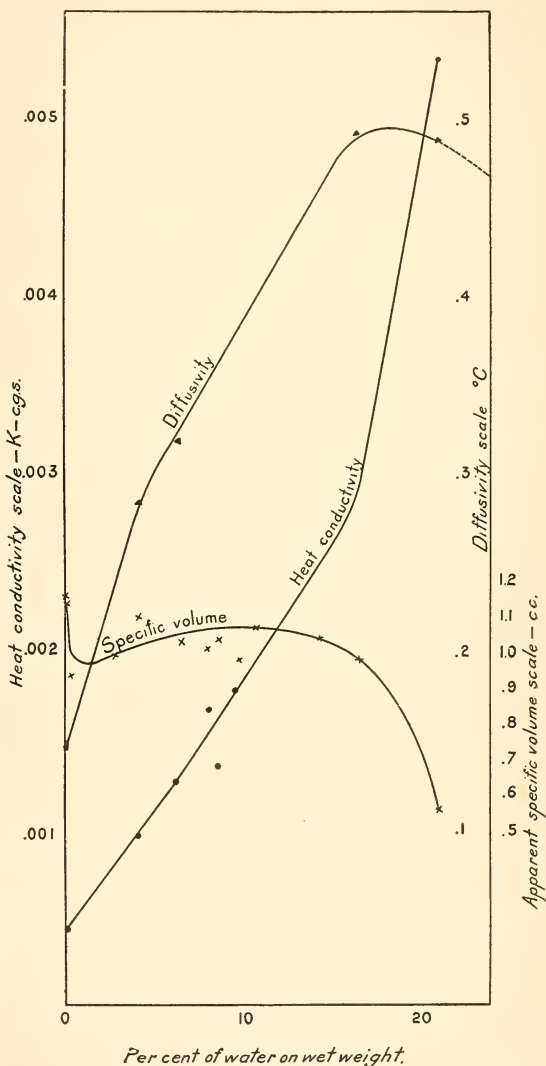


FIG. 14.—Curves for fine quartz powder similar to those in figure 13.

^a Kindly furnished by Brig. Gen. C. W. Raymond, U. S. A., retired, New York City.

TABLE VII.—*Mechanical analysis of Hudson River sand.*

Diameter of grain.	Conventional name.	Per cent.
<i>Millimeters.</i>		
2 - 1	Fine gravel.....	0.1
1 - 0.5	Coarse sand.....	0.5
0.5 - .25	Medium sand.....	1.3
.25 - .1	Fine sand.....	69.7
.1 - .05	Very fine sand.....	19.4
.05 - .005	Silt.....	7.6
.005 - .0	Clay.....	1.5

Figure 15 shows graphically the variation of the heat conductivity, diffusivity, and apparent specific volume as the moisture content of the soil increases. All the curves for Hudson River sand show the



FIG. 15.—Curves for Hudson River sand similar to those in figure 13.

same general form as those for coarse quartz in figure 13. The heat conductivity of the air-dry Hudson River sand is less than that of the coarse quartz and greater than that of the fine quartz flour. The highest value of the heat conductivity shown by Hudson River sand (in saturated condition) is below that of fine quartz flour and above that of coarse quartz. The maximum value of the diffusivity is, however, practically the same for each of these three soils; it comes at 16 per cent of moisture for fine quartz; at about 15.5 per cent for Hudson River sand; and at about 15.5 per cent for coarse quartz. It is possible, however, that this maximum diffusivity for Hudson River sand may be found at a lower moisture content when more experimental points are obtained; the value of the maximum, too, might be slightly higher.

PODUNK FINE SANDY LOAM.

The mechanical analysis of Podunk fine sandy loam is given in Table VIII; the values for $\frac{d\theta}{dt}$ and $\frac{d^2\theta}{dx^2}$ are found in Table III (p. 26) which also contains the apparent specific volume, moisture content, heat conductivity, and diffusivity. These values are obtained as shown above for coarse quartz.

TABLE VIII.—*Mechanical analysis of Podunk fine sandy loam soil.*

Diameter of grain.	Conventional name.	Per cent.
<i>Millimeters.</i>		
2 — 1	Fine gravel.....	0.0
1 — .5	Coarse sand.....	1.7
0.5 — .25	Medium sand.....	3.0
.25 — .1	Fine sand.....	47.4
.1 — .05	Very fine sand.....	38.6
.05 — .005	Silt.....	7.5
.005 — .0	Clay.....	1.8
Total..	100.0

Figure 16 shows the variation of the heat conductivity, diffusivity, and apparent specific volume with increasing moisture content for this soil. The heat conductivity curve here is very regular, and like that in figure 14 for fine quartz is concave, whereas the heat conductivity curve in figure 13 for coarse quartz is markedly convex and that for Hudson River sand, figure 15, is very slightly convex. The regularity of both the heat conductivity and diffusivity curves in figure 16 suggests that the wide departure of several points in figures 14 and 15 from the general trend of their respective curves is due to error. It is seen further that the heat conductivity and diffusivity of the air-dry Podunk soil are practically the same as those of the air-dry coarse quartz, both being higher than for the air-dry fine quartz (fig. 14). The heat conductivity of the Podunk soil increases more slowly with increasing water content than does the heat conductivity of fine quartz flour, but reaches a slightly higher final value than that of the fine quartz when both soils are in a saturated condition. The diffusivity curve for Podunk soil is similar to that for fine quartz.

LEONARDTOWN SILT LOAM.

The heat conductivity and diffusivity of Leonardtown silt loam at eight different moisture contents were determined in the same manner as for the preceding soils. The values of $\frac{d\theta}{dt}$ and $\frac{d^2\theta}{dx^2}$ are given in Table III (p. 27), together with the apparent specific volume, moisture content, heat conductivity, and diffusivity. The mechanical analysis of the soil is given in Table IX.

TABLE IX.—*Mechanical analysis of Leonardtown silt loam.*

Diameter of grain.	Conventional name.	Per cent.
<i>Millimeters.</i>		
2 -1	Fine gravel.....	1
1 -0.5	Coarse sand.....	2
0.5 - .25	Medium sand.....	3
.25 - .1	Fine sand.....	7
.1 - .05	Very fine sand.....	12
.05 - .005	Silt.....	58
.005- .0	Clay.....	15

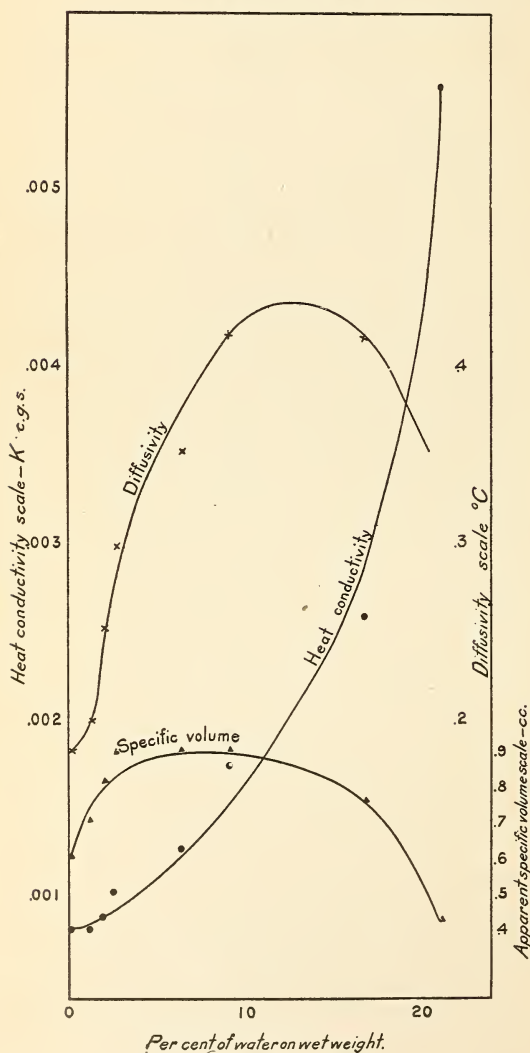


FIG. 16.—Curves for Podunk fine sandy loam similar to those in figure 13.

Figure 17 shows the variation of the heat conductivity, diffusivity, and apparent specific volume with increase in moisture content. Air-dry Leonardtown soil shows about the same heat conductivity as the coarse quartz (fig. 13), Hudson River sand (fig. 15), and Podunk soil (fig. 16). As the moisture content increases the heat conductivity of the Leonardtown soil rises a little faster than that of the Podunk soil or that of the fine quartz flour (fig. 14).

MUCK SOIL.

The experimental and calculated values used in deriving the heat conductivity and diffusivity of a muck soil containing 25 per cent of organic matter are given in Table III (p. 28), the experimental and analytical procedure being the same as used for the soils already discussed.

Figure 18 shows graphically the variation of the apparent specific volume, heat conductivity, and diffusivity of this muck soil with increase in moisture content. The absolute values of both the heat conductivity and diffusivity are in general lower than for the soils already treated. The great departure of some experimental values shown in this figure from the general trend is due to the difficulty of securing even distribution of moisture. The apparent specific volume curve is fairly regular, and where single values lie off the curve, the effect of this departure upon the heat conductivity would produce neither the magnitude of effect nor the direction of effect actually observed in the heat conductivity and diffusivity. Consequently,

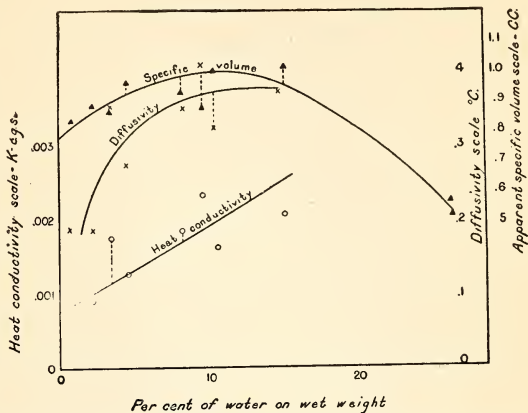


FIG. 17.—Curves for Leonardtown silt loam similar to those in figure 13.

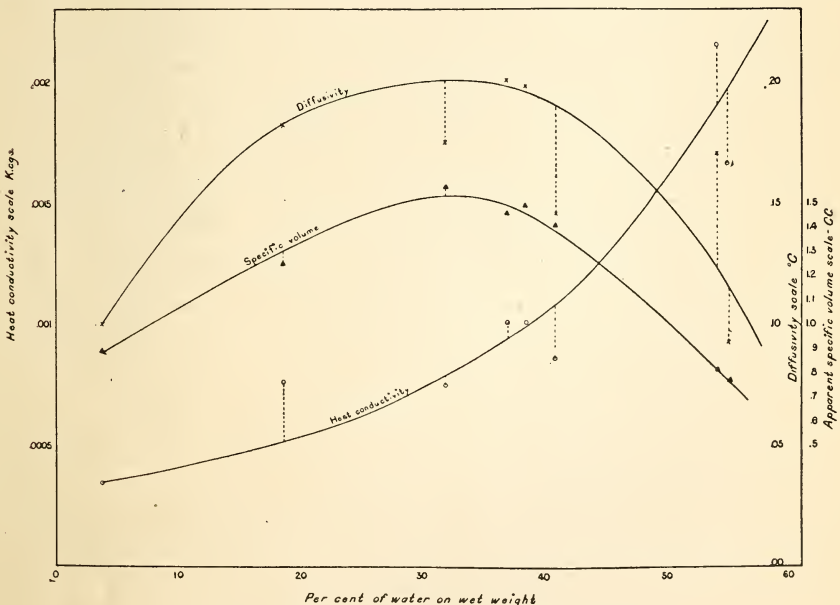


FIG. 18.—Curves for muck soil similar to those in figure 13.

the irregularity in these last is probably due largely to uneven distribution of moisture. It must be remembered, too, that the graphical calcu-

lation of $\frac{d^2\theta}{dx^2}$ may well produce such variations. The pronounced concavity of the heat conductivity curve and the convexity of the diffusivity curve are worthy of note. This question will be taken up under the general discussion of the results obtained in this investigation.

HAGERSTOWN LOAM, GALVESTON CLAY, AND NO. 4 SAND.

A summary of the experimental and derived data for the Hagerstown loam, Galveston clay, and No. 4 sand is contained in Table III (p. 28). The mechanical analyses of the two soils proper are given in Table X. No. 4 sand is an average sample of the fine sand from many different soils of the United States. This sample was obtained by mixing the No. 4 fractions of soil obtained in the regular routine mechanical analysis of a large number of soils. For the Hagerstown and Galveston clay soils one heat conductivity determination was made in an air-dry and one in saturated condition. These values compare very well with each other and with the values given for the soils considered above. The high value of the heat conductivity of the dry No. 4 sand is due to the close packing of its particles, evidenced by its low apparent specific volume. The experimental procedure and calculations were the same for these three soils as for the six soils given above.

TABLE X.—*Mechanical analyses of Hagerstown loam and Galveston clay soils.*

Diameter of grain.	Conventional name.	Hagerstown loam.	Galveston clay.
<i>Millimeters.</i>		<i>Per cent.</i>	<i>Per cent.</i>
2 - 1	Fine gravel.....	0.9	0.7
1 - 0.5	Coarse sand.....	3.0	1.5
0.5 - .25	Medium sand.....	2.1	0.4
.25 - .1	Fine sand.....	6.5	7.0
.1 - .05	Very fine sand.....	13.9	28.9
.05 - .005	Silt.....	54.3	50.1
.005 - 0	Clay.....	19.1	31.0
Total.....	99.8	99.6

TABLE XI.—*Fundamental data for determination of heat conductivity and diffusivity.*

[Name of soil: Norfolk sand.]

RATE OF RISE OF TEMPERATURE IN SOIL AT DIFFERENT DISTANCES FROM A STEADY SOURCE OF HEAT.

5 cm. from heat source.		10 cm. from heat source.		15 cm. from heat source.		20 cm. from heat source.		25 cm. from heat source.		30 cm. from heat source.	
Time.	Temp.	Time.	Temp.	Time.	Temp.	Time.	Temp.	Time.	Temp.	Time.	Temp.
<i>min. sec.</i>	<i>° C.</i>	<i>min. sec.</i>	<i>° C.</i>	<i>min. sec.</i>	<i>° C.</i>	<i>min. sec.</i>	<i>° C.</i>	<i>min. sec.</i>	<i>° C.</i>	<i>min. sec.</i>	<i>° C.</i>
0 0	20.5	0 0	20.3	0 0	20.4	0 0	20.5	0 0	20.4	0 0	20.4
7 0	20.7	12 30	20.4	15 30	20.6	21 30	20.9	22 0	20.7	22 30	20.7
12 0	20.9	20 30	20.7	26 30	21.0	35 0	21.4	31 30	21.0	32 0	21.0
15 45	21.2	26 30	21.0	38 30	21.7	44 15	21.9	39 10	21.2	45 0	21.3
23 15	22.3	38 0	21.8	49 30	22.3	52 30	22.2	48 10	21.6	53 30	21.8
25 45	23.0	47 0	22.5	63 0	23.1	67 30	23.0	53 0	21.9	60 0	22.0
28 10	23.5	57 15	23.6	85 45	24.8	86 15	24.0	59 30	22.1	87 15	22.8
30 0	24.0	66 20	24.7	97 30	25.6	95 15	24.5	68 0	22.4	96 0	23.0
34 0	25.0	85 20	26.8	111 0	26.5	87 0	23.3	107 0	23.4
37 30	26.0	93 15	27.8	124 30	27.5	118 30	25.5	98 30	23.8	119 10	23.8
40 0	26.9	100 15	28.6	141 30	28.5	142 0	26.6	112 5	24.2	134 15	24.1
43 0	27.7	110 45	29.7	173 0	30.1	173 15	27.9	125 15	24.8	148 45	24.5
46 30	28.7	128 30	31.4	193 0	31.2	193 30	28.8	143 0	25.3	174 0	25.2
48 50	29.4	135 40	32.0	221 30	32.5	222 0	29.8	173 40	26.4	205 0	26.0
54 0	30.9	147 30	33.0	240 45	33.4	252 30	30.9	184 45	26.8	222 45	26.5
57 0	31.7	155 0	33.6	267 20	34.4	290 15	31.9	211 0	27.6	2 2 0	27.0
62 0	33.1	172 30	34.9	290 0	35.0	324 40	32.5	241 45	28.5	2 8 30	27.8
66 0	34.0	183 15	35.6	324 30	35.7	253 15	28.8	312 30	28.3
70 0	35.0	203 0	36.9	290 40	29.8	325 30	28.4
85 0	38.3	220 30	37.8	312 0	30.0
91 0	39.5	240 15	38.7	325 0	30.2
96 30	40.4	251 45	39.2
105 20	41.9	289 15	40.5
113 0	43.0	324 0	41.2
122 15	44.3
140 30	46.3
154 30	47.7
172 0	49.2
192 0	50.6
208 30	51.6
231 0	52.8
251 30	53.5
289 0	54.6
324 0	55.1

Temperature of heat source 100°-99° C.

Heat applied at 0' 0".

Apparent specific volume 0.750.

Moisture content, wet weight, 0.1 per cent.

NORFOLK SAND. THERMOMETERS PLACED 5 CENTIMETERS APART.

Table XI contains the experimental observations for the temperature-time curves of air-dry Norfolk sand, the thermometers being

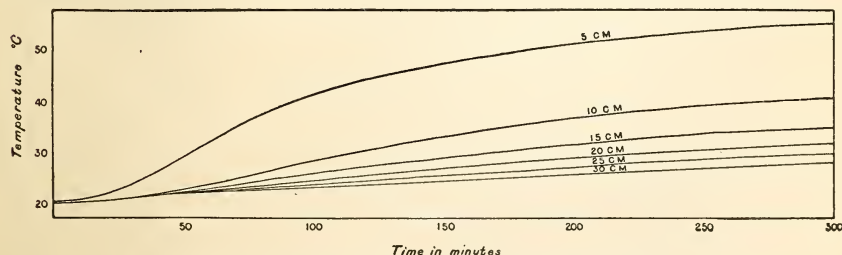


FIG. 19.—Temperature-time curves for air-dry Norfolk sand.

5 cm. apart. The mechanical analysis of this soil is given in Table XII. Figure 19 shows the basal temperature-time curve for the thermometers placed at 5, 10, 15, 20, 25, and 30 cm., respectively,

from the heat source and in the central diameter of the soil box. Figure 20 gives the temperature-time curve for the thermometer at 20 cm. from the heat source, but on a larger scale than in figure 19, in order to determine $\frac{d\theta}{dt}$, which is found to be $0.0487^\circ \text{ C. per minute}$. This rate of rise begins at 27 minutes after applying the soil box to the heat source, consequently a cross section of the curves in figure 19 should be taken at this time in order to find the value of $\frac{d^2\theta}{dx^2}$. But the temperature gradient is so flat at this distance from the heat

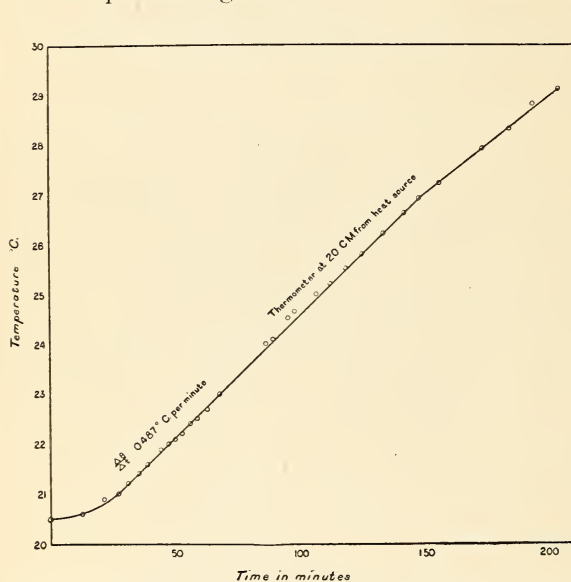


FIG. 20.—Temperature-time curve for air-dry Norfolk sand, No. 4 thermometer plotted to larger scale.

source that $\frac{d\theta}{dx}$, the first differential, can not be measured accurately with thermometers reading only to 0.1° C. , much less a second differential $\frac{d^2\theta}{dx^2}$. This makes clear the necessity of inserting thermometers at short distances from each other, and using a steep temperature fall, if the value $\frac{d^2\theta}{dx^2}$ is to be obtained even very roughly. As a makeshift, the temperature gradient at 50 minutes was taken from the data given in Table XI, plotted in figure 21 and a value of $\frac{d^2\theta}{dx^2}$ found graphically as 0.0325° C. Using the formula

$$\left[\frac{C}{V} \cdot \left(\frac{d\theta}{dt} \right) \div 60 \right] \div \frac{d^2\theta}{dx^2} = K$$

it is seen that $K=0.00615$, since V , the apparent specific volume is 0.750 ; $C=0.1848$, the effective specific heat, and $\frac{d\theta}{dt}=0.0487^\circ \text{ C. per minute}$. This value of $\frac{d^2\theta}{dx^2}$ at 50 minutes is obtained using 2.5 cm. as the value of x . A similar gradient taken at 75 minutes after the soil box is applied to the heat source gives a value for $\frac{d^2\theta}{dx^2}$ of

0.0096° C. Now the rate of rise of thermometer No. 4 at 75 minutes is the same as at 50 minutes, consequently this value 0.0096 for $\frac{d^2\theta}{dx^2}$ when introduced into the above equation gives a value for the heat conductivity over three times greater ($K=0.0211$) than that obtained using the $\frac{d^2\theta}{dx^2}$ obtained from the temperature gradient at 50 minutes.

This large error emphasizes the necessity of taking the temperature gradient, and the rate of rise of the chosen thermometer (No. 4 in this case) at the time when the rate of rise of temperature just begins to be constant. When this is done a comparison of the heat conductivity values shows regularity which otherwise disappears. It is clear from the values of the heat conductivity of the various air-dry soils given above that both of these values just given ($K=0.00615$ and $K=0.0211$) for air-dry Norfolk sand are very much too high. The lower values given in the work on the foregoing nine soils, moreover, are remarkably close to those obtained by other investigators, as will be shown farther on.

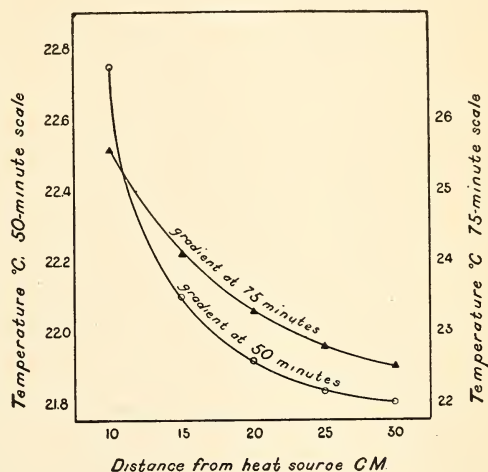


FIG. 21.—Temperature gradients for air-dry Norfolk sand at 50 minutes and 75 minutes.

TABLE XII.—Mechanical analysis of Norfolk sand

Diameter of grain.	Conventional name.	Per cent.
<i>Millimeters.</i>		
2 - 1	Fine gravel.....	0.2
1 - 0.5	Coarse sand.....	6.3
0.5 - .25	Medium sand.....	22.2
.25 - .1	Fine sand.....	63.7
.1 - .05	Very fine sand.....	4.1
.05 - .005	Silt.....	1.0
.005 - .0	Clay.....	1.8

CARBORUNDUM—EFFECT OF SIZE OF GRAIN UPON HEAT CONDUCTIVITY AND DIFFUSIVITY.

Table XIII contains the values $\frac{d\theta}{dt}$ and $\frac{d^2\theta}{dx^2}$ for nine different grades of carborundum powder, whose respective moisture contents, apparent specific volumes, average diameter of grain, and effective specific heats are given under appropriate headings, as well as the calculated values of the heat conductivity and diffusivity. In figure 22 the apparent specific volume, heat conductivity, and diffusivity of the

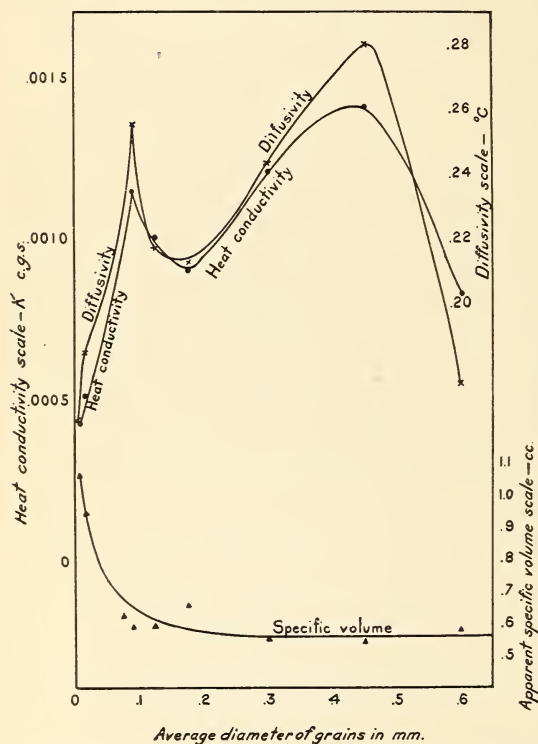


FIG. 22.—Curves showing the effect of size of grain of carborundum powder upon apparent specific volume, heat conductivity, and diffusivity.

is seen, too, in the powders which lie between 0.1 and 0.2 mm. average diameter of grain. The apparent specific volume in these two cases lies higher than might be expected from the curve drawn through the first four points, and we should accordingly expect the heat conductivity and diffusivity to be lowered by the looser packing of powder thus indicated. This is exactly the case, and at the point where the powder has a grain diameter of 0.3 mm. it is seen that the heat conductivity and diffusivity have again risen, while the apparent specific volume—as it should be—is lower

carborundum powders are plotted as a function of the average diameter of the grain of each powder. Diameters are taken as abscissas; and as ordinates, the heat conductivity (to the left), the diffusivity (to the right, above), and the apparent specific volume (to the right, below).

The curve for the apparent specific volume shows a drop as the size of grain increases. The heat conductivity and diffusivity curves both show the effect of this, as they rise steeply to values about 30 per cent higher than those for the finest grade of carborundum. This influence of packing

TABLE XIII.—Effect of size of grain upon heat conductivity, for air-dry carborundum powder. *Summary of experimental and derived data.*

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)
Apparent specific volume V_s .	Moisture content on wet weight.	Moisture content on dry weight.	Effective specific heat, C .	Rate of rise of temperature at 5 cm. from heat source (No. 5 thermometer). $\frac{d\theta}{dt}$	Temperature at foot of thermometer straight line rise.	Time at foot of thermometer straight line rise.	Period required for rise of 6.3° C. at 5 cm. from heat source.	Rate of change of temperature gradient at 5 cm. from heat source for $dx=0.5$ cm. $\frac{d^2\theta}{dx^2}$	$(4) \times (5) =$ calories per gram of soil per minute.	$(10) \div (1) =$ calories per cubic centimeter per minute.	$(11) \div (60) =$ calories per cubic centimeter per second.	$(12) \div (9) =$ calories per square centimeter per second per degree change in temperature gradient $= K = \frac{C(d\theta/dt) \div (d^2\theta/dx^2)}$	$(5) \div (9) =$ diffusivity rate of rise of temperature at 5 cm. from heat source per degree change in temperature gradient. $\frac{(d\theta/dt)}{(d^2\theta/dx^2)}$	Conventional name of grade.	Average diameter of grain.
cc.	Per cent.	Per cent.	Calories.	°C per min.	°C.	min. sec.	min. sec.	°C. per cm.					°C. per min.		mm.
0.584	0.0216	0.0217	0.1659	0.454	26.7	16 30	24 30	2.6	0.0753	0.1289	0.00214	0.0008231	0.1746	40	0.60
.518	.0052	.0052	.1659	.447	27.35	14 45	24 40	1.6	.0742	.1350	.00225	.001406	.2800	50	.45
.559	.0075	.0075	.1659	.389	26.6	17 40	27 55	1.6	.0646	.1153	.001923	.001202	.2431	60	.30
.659	.0076	.0076	.1659	.468	21.9	17 10	24 35	2.2	.0777	.1180	.001965	.000832	.2127	90	.175
.601	.0039	.0039	.1659	.477	22.0	15 12	24 18	2.2	.0791	.1317	.002194	.000997	.2170	150	.125
.596	.00393	.00393	.1659	.460	13.4	18 12	26 25	1.8	.0731	.1227	.00206	.00114	.2555	180	.090
.625	.0166	.0167	.1659	.396	19.4	20 18	29 40	1.8	.0657	.1050	.00175	.000975	.2200	220	.075
.945	.086	.087	.1659	.407	21.6	20 50	29 55	2.2	.0675	.0715	.001124	.0005110	.1850	F F F	.015
1.066	.0323	.0324	.1659	.294	22.8	21 20	36 00	1.8	.0488	.0458	.000763	.000421	.1633	60'	.006

and seems to have reached its normal value. From these results it is seen that in air-dry condition No. 50 carborundum powder of average grain diameter 0.45 mm. shows the maximum heat conductivity and diffusivity and the minimum apparent specific volume. The effect of moisture films upon the carborundum surfaces is of course to increase the heat conductivity and diffusivity by securing better contact between the grains; but in this case the No. 50 carborundum powder showed about one-fourth of the moisture content given by the other coarse grade powder No. 40, and 30 per cent less than No. 60 or No. 90; consequently this maximum shown by No. 50 powder is not due to moisture.

If this maximum heat conductivity of a powder having grains 0.45 mm. in diameter be applied in the case of an air-dry soil, one may expect to find the best conduction in the case of a medium sand whose grains are officially classified as ranging between 0.5 and 0.25 mm. in diameter.

For assistance in making experimental observations and calculations, acknowledgments are due to Mr. J. M. Jester.

GENERAL DISCUSSION OF RESULTS.

The transfer of heat from one soil particle to another is influenced by the fluid filling the space between them, and of course this transfer is more easily effected when the fluid is a good conductor of heat. In addition to the heat conductivity of the fluid, another factor has to be considered; that is, the resistance to the heat transfer which is found at the boundary between the substances in contact. This "transfer resistance," or its reciprocal, "transfer conductivity," is of sufficient magnitude to enter into thermal calculations.

If the fluid be in motion, there will be developed a resistance to the transfer of heat which has been shown to vary approximately with the square root of the velocity of the fluid. The transfer conductivity for the fluids air and water may be roughly compared as follows:

When heat passes from metal to air, the transfer conductivity is given by the equation,

$$K = 0.000,028(2 + \sqrt{v}),$$

where v is the velocity with which the air passes the surface of the metal.

For the passage of heat from water to metal, this transfer resistance is much less, as shown by the greater transfer conductivity,

$$K = 0.000,028(300 + 180\sqrt{v}).$$

Assuming $v = 1$ cm. per second in each case, K for air to metal is 0.000,084; while for water to metal K is 0.01344; that is, the

heat passes from water to metal about sixteen hundred times more readily than from air to metal, at this unit velocity of 1 cm. per second.

For a soil: Assume (1) that the velocity of soil atmosphere in a dry soil is very nearly zero; and (2) that the velocity of soil water in a wet soil be nearly zero; and (3) that the transfer conductivity of soil material to gas and to water has the same ratio as that which holds for the transfer conductivity of a metal to gas and to water.^a

Then the above equations indicate that heat will pass from a soil grain to soil water one hundred and fifty times easier than from a soil grain to soil atmosphere. This comparison seems to point out one reason why an air-dry soil shows such a low heat conductivity. The rapid circulation of soil atmosphere as eddy currents within the minute spaces between the soil grains would, however, decrease very greatly the transfer resistance. Such eddy currents are inevitably set up as the hot-soil atmosphere on one side of a soil cavity expands and moves up and is replaced by the colder gases.

Consider the air-dry soils: It is seen in Table XIV that where the apparent specific volume (the packing) is about the same the heat conductivities are very nearly the same. Fine quartz flour shows about one-half the conductivity of the other air-dry soils, but this is due in part to its high apparent specific volume. Comparing the moisture film expressed as grams water per square centimeter of soil grain area, it is seen that the coarse quartz has a film of water some nine times heavier than that on the grains of the fine quartz flour, and from this one would expect better contact between the grains of coarse quartz and better heat transfer, as is the case. This good contact between soil grains may also be secured by the smaller sizes of grains wedging into the spaces between the larger sizes. Thus, Leonardtown silt loam shows a higher heat conductivity than Podunk fine sandy loam, although the moisture film per square centimeter on the Podunk soil grains is heavier than that on the Leonardtown soil grains. The Leonardtown soil contains more clay and silt, and these assist in securing the transfer of heat through the soil.

^a See Preston, "Theory of Heat."

TABLE XIV.—*Data showing the relation between area, moisture film, and heat conductivity and diffusivity of air-dry soils.*

Name of soil.	Moisture content.	Apparent specific volume.	Area per gram.	Weight of water per sq. cm. of soil surface.	Heat conductivity <i>K</i> .	Diffusivity, rise per minute, 1 degree gradient.
	<i>Percent (on wet weight).</i>	<i>cc.</i>	<i>sq. cm.</i>	<i>Grams.</i>	<i>C. g. s. units.</i>	<i>°C.</i>
Coarse quartz.....	0.065	0.601	97	0.000,009,8	0.000917	0.173
Fine quartz.....	.083	1.130	740	.000,001,12	.000407	.144
Hudson River sand.....	.256	.738	345	.000,007,1	.000654	.1613
Podunk fine sandy loam.....	.267	.698	415	.000,006,43	.000792	.179
Leonardtown silt loam.....	.80	.828	1,892	.000,004,2	.000882	.185
Muck soil.....	3.78	.895	(?)6,000	.000,006,3	.000349	.0999
No. 4 sand.....	.018	.648	137	.000,001,3	.00075	.159
Hagerstown loam.....	1.11	.949	2,270	.000,004,4	.000699	.199
Galveston clay.....	1.39	.949	3,280	.000,004,24	.000577	.149
Norfolk sand.....	.1	.750	300	(?).000,003,3	(?).006	(?)1.5

These values for the area of each soil per gram as shown in Table XIV are, of course, very rough. They were calculated from the mechanical analyses given above for each soil, assuming the soil grains to be spheres. Still these rough values, when used with the moisture contents, serve to give a picture of the probable dimensions of the film of water surrounding the soil grains and making contact between them, and thus afford a better idea of the relation of soil moisture to heat conductivity.

For the muck soil no mechanical analysis was made, since it would be meaningless. The area per gram was taken as a very rough approximation by comparing the heat given out on wetting this soil with the heat evolved by other soils, whose mechanical analysis is available, as shown in Table IV. It is interesting to note that using this extremely crude method of comparison the moisture film upon the muck soil appears to be of the same order of magnitude as the films on the other soils.

The heat conductivity of water as given by Lees^a is 0.00149 (metric) for *K*. Consequently the heat conductivity of soil and water mixtures eventually must approach this value as the percentage of soil in the water decreases. The increase in heat conductivity of a soil produced by wetting it is then entirely due to the better contact between the soil grains thus produced, since the soil material has in continuous massive condition a much higher heat conductivity than water.

When the heat conductivity of soil increases (on wetting) as a nearly linear function of the moisture present, it means that the good contact secured between the soil grains by the soil moisture has an effect enough greater than linear to make up the decrease in heat conductivity which is produced by the addition of water—a sub-

^a Phil. Mag. (5), 49, 289 (1900).

stance whose heat conductivity is approximately one-seventh ^a that of the soil material when compacted into rock. Thus, coarse quartz powder when saturated with water (20 per cent on the wet weight) shows a lower heat conductivity than finer-grained soils at the same moisture content, since a larger fraction of the water is held by the coarse quartz in larger intergrain cavities where the heat conductivity of water is a greater factor in the heat transfer than the function of the water in securing contact between the quartz grains.

The diffusivity curve for coarse quartz goes through a maximum at 5.5 per cent of moisture, whereas the maximum apparent specific volume of coarse quartz comes at a lower moisture content—about 3 per cent—as shown in figure 13. This same lack of agreement between optimum water content and maximum diffusivity is seen in figure 14 for the fine quartz flour, which has a maximum apparent specific volume at about 11 per cent of moisture and a maximum diffusivity at 16 per cent of moisture (calculated on the wet weight). Inspection of figures 15, 16, 17, and 18 shows that this is a general characteristic of widely differing soil types; i. e., the optimum water content of each soil studied lies below the percentage of water which gives to the soil its highest diffusivity.

The reason for this is that at the optimum water content we have the soil grains formed into larger aggregates and the spaces between these aggregates act as an air insulation against the passage of heat. As more water is added some of these soil grain aggregates break up and their constituent grains now serve to fill in the spaces between the larger aggregates, and the added water gives still better contact between the soil crumbs and the small grains. Thus the total effect is a better conduction of heat and a faster rise of temperature. But when water is added to the soil over a certain percentage (which is different for each soil) the temperature of the soil will rise more slowly, although heat is being conducted by the soil better than at a lower moisture content. This effect is produced by the high heat capacity of water, which is almost five times that of a dry soil.

For purposes of comparison and for convenient reference the heat conductivities of water, ice, rocks, and soils, together with the sources of the data and names of the experimenters, are given in Table XV. ^b The conductivities headed "Eng. K " are the values of K to be used in the formula $q = \frac{K a t}{x}$, where q is the heat, in British thermal units, which will pass between the bases of a right prism

^a For sandstone, $K=0.01066$, according to Forbes in Tait on Heat.

^b This set of conductivities was kindly furnished us by Mr. S. H. Woodard, resident engineer of the East River Division, Pennsylvania Tunnel and Terminal Railroad Company.

"*a*" square feet in area, *x* feet apart and at a difference of *t* degrees Fahrenheit in *one hour*. The conductivities headed "*Metric K*" are the values to be used for "*K*" in the same formula when all factors are in the C. G. S. units, and *q* then represents calories in *one second*; *a*, area in square centimeters; *t* the temperature difference in degrees centigrade; and *x* the distance, between the bases of the prism of reference, in centimeters.

TABLE XV.—*Heat conductivities.*

Material.	Eng. K.	Metric K.	Reference.	Experimenter.
Water.....	0.33867	0.00140	Philos. Mag., vol. 49.....	Chas. H. Lees.
Do.....	.36044	.00149do.....	Do.
Do.....	.35803	.00148do.....	Do.
Do.....	.32899	.00136	Royal Soc. London, vol. 191.....	Do.
Do.....	.32899	.00136	Barker's Physics.....	Weber.
Do.....	.48381	.0020	Glazebrook's Heat.....	Forbes.
Ice.....	1.3789	.0057	Barker's Physics.....	Neumann.
Do.....	.53945	.00223	Science Lectures at So. Kensington, vol. II, p. 25.	Forbes.
Do.....	.55638	.0023do.....	Lucien de la Rive.
Sand.....	1.7901	.000740	Royal Soc. London, V. 65, p. 285.....	Lamb and Wilson.
Do.....	.62895	.0026	Glazebrook's Heat.....	Forbes.
Do.....	.63379	.00262	Science Lect. at So. Kensington.....	Do.
Do.....	.63137	.00261	Carpenter's Heating and Ventilation.....	Do.
Sandy soil.....	<i>a</i> .660	.00271	Tait on Heat.....	Neumann.
Sandy loam.....	3.28992	.0136	Am. Acad. Arts and Sci., vol. 38, No. 23, May, 1903.	Everett.
Greenwich Park gravel (damp).....	3.02382	.0125do.....	Forbes.
Sandstone.....	2.58355	.01068	Sci. Lect. So. Kensington.....	Do.
Do.....	<i>a</i> 2.58	.01066	Tait on Heat.....	Herschell, Lebour, and Dunn. ^b
Sandstone and hard dry grit.....	1.33048	.0055	Am. Acad. Arts and Sci. vol. 38, No. 23.	Kelvin.
Craigleith Quarry sandstone (damp).....	2.58839	.0107do.....	Peclet.
Limestone:				Do.
Fine-grained.....	1.2228	.005055	Reck.....	Do.
Coarse-grained.....	.87352	.003611do.....	Do.
Porphyritic trachyte.....	1.4246	.00589	Carpenter's Heating and Ventilation.....	Forbes.
Trap.....	1.0039	.00415	Sci. Lect. So. Kensington.....	Do.
Do.....	1.00802	.004167	Carpenter's Heating and Ventilation.....	Do.
Trap:				
Density=2.90.....	.74991	.0031	Amer. Acad., vol. 38, No. 23, May, 1903.	B. O. Pierce.
Density=2.82.....	.87086	.0036do.....	Do.
Whinstone trap rock.....	.67733	.0028do.....	H. L. and D. ^b
to	to	todo.....	Kelvin.
Calton Hill trap (damp).....	1.16115	.0048do.....	Do.
Amygdaloid:	1.01600	.0042do.....	Do.
Density=2.67.....	.84667	.0035do.....	B. O. Pierce.
Density=2.71.....	.82248	.0034do.....	Do.
Conglomerate:				
Density=2.55.....	1.13695	.0047do.....	Do.
Density=2.64.....	1.25791	.0052do.....	Do.
Tough dry clay.....	.53219	.0022do.....	H. L. and D. ^b
Devonshire clay slate.....	.65315	.0027do.....	Do.
Serpentine.....	1.42724	.0059do.....	Neumann.
Cornish red serpen- tine.....	1.06438	.0044do.....	H. L. and D. ^b
Caen stone.....	1.04019	.0043	Am. Acad., vol. 38, No. 23, May, 1903.	Do.
Chalk.....	.48381	.0020do.....	Do.
to	to	todo.....	Do.
to	.79828	.0033do.....	Do.
Marble.....	1.71753	.00710	Philos. Mag., vol. 49.....	Chas. H. Lees.
Marble, fine grained, gray.....	2.3385	.009667	Reck.....	Peclet.
Marble, coarse grain- ed, white.....	1.86751	.00772do.....	Do.
Marble.....	1.52401	.0063	Wiedemann's Annalen, bd. 14, 1881, p. 23.	C. Christiansen.
Marble, fine grained.....	1.69334	<i>c</i> .007	Amer. Acad.....	Peirce and Willson.
Marble, coarse grained.....	1.33048	.0055do.....	Do.
Carrara white marble.....	1.23372	.0051	Vol. 38, No. 23.....	Do.

^a Given.^b For British Association.^c Average of fifteen experiments.

TABLE XV.—*Heat conductivities*—Continued.

Material.	Eng. K.	Metric K.	Reference.	Experimenter.
American and Italian colored marbles.	1.49982 to 1.83848	0.0062 to .0076	Vol. 38, No. 23.....	Peirce and Willson.
Sugar-white marble.	1.86267	.0077	do.....	Despretz.
Marble, fine grained, gray.	2.34648	.0097	do.....	Do.
English marbles and limestones.	1.1370 to 1.35467	.0047 to .0056	do.....	H. L. and D. ^a
Granite.....	1.28210	.0053	Amer. Acad., vol. 38, No. 23.....	Do.
Glass.....	.67008	.00277	Am. Acad. Arts and Sci.....	Peirce and Willson.
Do.....	.55101	.002278	Carpenter's Heating and Ventilation..	Peclet.
Do.....	.50389	.002082	Reek.....	Do.
Do.....	.120953	.0005	Glazebrook's Heat.....	Forbes.
Do.....	.59267	.00245	Royal Soc., London.....	Chas. H. Lees.
Glass, English plate..	.55638	.0023	Am. Acad., vol. 38, No. 23.....	H. L. and D. ^a
Glass, American plate	.62895 to .67733	.0026 to .0028	do.....	Peirce and Willson.

^a For British Association.

Likewise the results of Hutton and Beard ^a are given in Table XVI, the heat conductivities being calculated in the metric system, and thus corresponding to the values given in Table XV under the heading "Metric K."

Table XVII contains a similar set of data taken from J. W. Richards' Metallurgical Calculations.^b Here, too, the conductivities are given in the metric units.

TABLE XVI.—*Heat conductivities.*

[Results of Hutton and Beard.]

Material.	Metric K.	Material.	Metric K.
Sand, white Calais.....	0.00060	Lime.....	0.00029
Carborundum, fine.....	.00050	Magnesia, fused.....	.00047
coarse.....	.00051	"Mabor" brick.....	.00050
Quartz, "enamel".....	.00036	calcined Greek.....	.00045
fused.....	.00039	calcined "Veitsch".....	.00034
Fire brick.....	.00028	Pattinson's light calcined....	.00016
Retort graphite.....	.00040	Kieselguhr (infusorial earth).....	.00013

^a Electrochem. and Met. Ind., 3, 201 (1905).^b Electrochem. and Met. Ind., 4, 102 (1906).

TABLE XVII.—*Heat conductivities.*

[From J. W. Richards' Metallurgical Calculations.]

Material.	Metric K.	Material.	Metric K.
Ice (datum useful in refrigerating plants, where pipes become coated with ice, as in Gayley's method of drying blast).....	0.00500	Infusorial earth (0°-650°).....	0.00038
Snow.....	.00050	Clinker, in small grains (0°-700°).....	.00110
Glass (10°-15°).....	.00150	Coarse ordinary brick dust (0°-100°).....	.00039
Water.....	.00120	Chalk (0°-100°).....	.00028
Quartz sand (18°-98°).....	.00060	Wood ashes (0°-100°).....	.00017
Carborundum sand (18°-98°).....	.00050	Powdered charcoal (0°-100°).....	.00022
Silicate enamel (20°-98°). (Explains the small conductance of enameled iron ware).....	.00040	Powdered coke (0°-100°).....	.00044
Fire-brick dust (20°-98°).....	.00028	Gas retort carbon, solid (0°-100°).....	.01477
Retort graphite dust (20°-100°). (Datum useful where articles are packed in this poorly conducting material).....	.00040	Cement (0°-700°).....	.00017
Lime (20°-98°). (Datum would be highly useful for oxyhydrogen platinum furnaces, if it were only known at high temperatures).....	.00029	Alumina bricks (0°-700°).....	.00204
Magnesia brick, dust (20°-100°).....	.00050	Magnesia bricks (0°-1,300°).....	.00620
Magnesia calcined, Grecian, granular (20°-100°).....	.00045	Fire bricks (0°-1,300°).....	.00310
Magnesia calcined, Styrian, granular (20°-100°).....	.00034	Fire bricks (0°-500°).....	.00140
Magnesia calcined, light, porous (20°-100°).....	.00016	Marble, white (0°).....	.0017
Infusorial earth (Kieselguhr (17°-98°).....	.00013	Pumice.....	.0006
		Plaster of Paris.....	.0013
		Felt.....	.000087
		Paper.....	.00040
		Cotton.....	.000040
		Wool.....	.000035
		Slate.....	.00081
		Lava.....	.00008
		Pumice.....	.00060
		Cork.....	.00072
		Pine wood.....	.00047
		Oak wood.....	.00060
		Rubber.....	.00047

